Volume 1 *November 1931* *Number 5*

PHYSICS

A STUDY OF LIGHT SIGNALS IN AVIATION AND NAVIGATION

By I. Langmuir and W. F. Westendorp
General Electric Company, Schenectady, N. Y.

(Received September 16, 1931)

Abstract

Visibility of point sources.—Laboratory experiments have been devised to make measurements of the visibility of light signals under conditions essentially similar to those encountered by the aviator or the navigator. Data have been collected on the direct visibility of flashing point sources of light of different colors, flash lengths and intervals, against different backgrounds; the time it takes to locate a visible beacon was studied as a function of the beacon intensity and frequency of flashing. The threshold candle power C required for visibility of a point source at distance D (cm) against a background of brightness H (candles$\times$cm^{-2}) is given by the empirical equation $C/D^2 = 3.5\times10^{-9}\,H^{1/3}$. Colored point sources were not found to be useful except in the case of red lights with background intensities above moonlight. For an airplane approaching a beacon it is advantageous to use frequencies of flashing as high as 12 to 30 per minute, although with exceptionally clear atmosphere, lower frequencies may be better.

Visibility of diffuse light.—In a study of the visibility of flashes of diffuse light superimposed on a steady white background, white light flashes gave the best results. The sensitivity of the eye to light from point sources is from 10,000 to 170,000 times as great as from diffuse sources, this range corresponding to an increase in background brightness from 0.1 starlight up to moonlight. A selective differential photoelectric receiver is described which detects signals of *modulated diffuse light* of an intensity of only 4×10^{-11} candles$\times$cm^{-2}. This sensitivity is independent of the steady background brightness up to 100 times moonlight, and is from 6 to 13,000 times as great as that of the eye in the range of background intensity from darkness up to moonlight.

Diffusion of light in fog.—The greatest difficulty in transmission of light signals through fog lies in the loss of advantages of the point source. Dense fog may increase the distances at which *diffuse light signals* may be detected. The range depends to a considerable extent on the reflectivity of the ground. A theoretical treatment of the diffusion of light through fog, based on the scattering of the light rays by fog particles, indicates that airplanes can be guided through fog at distances of several miles by means of diffuse modulated light acting on a differential photoelectric receiver.

IN ALL forms of traffic, whether it be navigation, automobilism or simply walking, we depend on the use of our eyes. But most strikingly is this true for aviation. At present the pilot depends on direct vision for guiding his plane, and hence "visibility" is a vital factor in aviation. Methods have been developed recently by which "blind" flying is possible and practical for definite commercial routes. The extent to which these methods will prove useful cannot be predicted, but it is certain that light beacons and direct visibility will continue to be of great importance to the aviator.

Very little systematic experimental work seems to have been done previously on the visibility of light signals as used in aviation. The purpose of this paper is to present some results we have recently obtained.

The most obvious method to obtain data on the visibility of light beacons is to set up different types of beacons and to observe them from an airplane under various conditions. However, this method lacks a fundamental quality of most scientific investigation—that of arbitrary control of all conditions: the weather would be different on different nights, and, furthermore, even during a single night the background illumination due to moon or starlight would vary.

We have, therefore, adopted the plan of making measurements on visibility in a dark room in the laboratory where there is complete control over the illumination of the background, the location, the color and the flashes of the beacon, or other light source. It has been our aim to duplicate as far as possible in the laboratory the conditions of visibility encountered by the aviator or navigator, especially those characteristic of poor visibility.

In reasonably clear weather, when the direct rays from a beacon can reach the eyes of the aviator, he perceives the signal as a *flash* of *light* from a *point source*. The visibility of such point sources depends on many factors, such as the candle power of the source, its distance from the observer, the absorption or scattering of light by the intervening atmosphere, the color of the light, the diffuse background illumination (starlight, moonlight or sun-light), the duration and the frequency of the flashes, the presence of other disturbing point sources of light, and finally, upon the qualities of the observer: his eyesight, his mental condition, and especially upon his attentiveness.

Under conditions of poor visibility, particularly those due to the presence of fog, the light from a beacon or other source may be *scattered* to such an extent that the direct rays are not visible, although diffuse light from the source may still reach the aviator in perceptible amount.

The visibility of flashing diffuse light signals depends primarily upon the brightness of illumination in the fog surrounding the aviator produced by the signal, the color of the light, the intensity of any steady illumination of the fog (starlight, moonlight, etc.), and the duration and frequency of the flashes of the signal.

We shall see that the eye is relatively insensitive to diffuse light as compared to that from *point sources*. On the other hand, it has been possible by use of a photoelectric cell with amplifier equipment to detect *diffuse* light many thousand of times too faint to see with the eye.

Any estimate of the distance to which diffuse light signals of this kind can be transmitted involves a study of the laws of scattering of light by fog. We shall see that under certain conditions signals may be transmitted to relatively great distances through fog.

The aviator will be particularly interested in the time required to detect or locate a visible light signal. If a point source is of such low intensity (threshold intensity) that it is barely visible when pointed out to the observer, it takes an indefinitely long time for the observer to locate the source if it is in

an unknown position in a large uniformly illuminated background. The threshold intensity for a flashing point source is that which is required to render the flash barely visible when occurring at regular intervals and in a known position. If the flashes are then made to come at irregular intervals or in unknown positions, a relatively long time may be required to become aware of them.

A useful signal to the aviator must be recognizable within a very short time and must therefore be of an intensity appreciably greater than the threshold.

Visibility of Point Sources

When the light from a steady point source falls on the eye of the observer, it is focussed by the lens of the eye upon a very small area of the retina. The amount of light so focussed is proportional to the area of the pupil opening of the eye and to the light flux density, F, in lumens per cm^2. In clear air, without scattering or absorption,

$$F = C/D^2 \tag{1}$$

where C is the candle power of the source in the direction towards the observer, and D is the distance from the source to the observer.

In the presence of haze or fog the direct light flux from a point source decreases exponentially with increasing distance so that in general we have

$$F = (C/D^2)e^{-D/\lambda} \tag{2}$$

where $1/\lambda$ is the scattering or absorption coefficient characteristic of the fog or haze, and λ may be looked upon as the *mean free path* of the light rays.

A discussion of the magnitude of λ under various conditions of haze, for light of different colors, has been given by F. Benford.[1] It is of the same order of magnitude as the distance termed "visibility" by the aviator and thus except under conditions of fog is ordinarily to be measured in miles.

In our laboratory experiments we have not studied the effect of haze in decreasing the light flux received from a source. We have, however, determined under various conditions the visibility of light sources which produce known flux intensities F at the eye of the observer. These data are obviously applicable to the conditions encountered by the aviator if λ is known or if the exponent D/λ is small compared to unity, i.e., if λ is large compared to the distance D.

The visibility of a point source depends to a very great extent upon the brightness of illumination of the background. The aviator may encounter background intensities which range from that produced by bright sunlight on snow down to that of starlight on a cloudy night with forest covered ground.

When light from a point source, giving a flux intensity F in accord with Eq. (1), falls perpendicularly upon a plane surface of a perfect diffuse reflector, the brilliancy of illumination H of the surface in candles per cm^2 is

[1] F. Benford, Gen. Elec. Rev. **29**, 873 (1926).

$$H = F/\pi. \tag{3}$$

In Table I are given values of F and H characteristic of various sources. These light intensities[2] are those that would be observed at sea level when the point source is at the zenith of an exceptionally clear sky. Stars of the sixth magnitude corresponding to $F = 10^{-12}$ lumens $\times$ cm^{-2} are the faintest that can ordinarily be seen with the unaided eye.

TABLE I. *Light flux density F and brightness of illumination H produced by various sources.*

Source	F lumens$\times$cm^{-2}	H candles$\times$cm^{-2}	Brightness of source candles$\times$cm^{-2}
Sun	10.4	3.3	150000.
Full Moon	2.2×10^{-5}	7.1×10^{-6}	0.37
Venus (max. brightness)	1.1×10^{-8}	3.5×10^{-9}	—
1st magnitude star	8.5×10^{-11}	2.7×10^{-11}	—
6th magnitude star	8.5×10^{-13}	2.7×10^{-13}	—
Average starlight	1.8×10^{-7}	5.8×10^{-8}	5.8×10^{-8}
1 candle at 1 kilometer	10^{-10}	3.2×10^{-11}	—

Effect of flashing

If the light source consists of separate flashes of candle power C, each of which lasts for a time t_0, the candle power required for threshold visibility will in general be greater than that required for a steady light.

Blondel and Rey[3] have found that the threshold intensity F required for visibility of a light of duration t_0 seconds is given by

$$Ft_0 = F_\infty(t_0 + 0.21) \tag{4}$$

where F_∞ is the threshold intensity for a steady light. Thus when a flash lasts several seconds so that 0.21 is negligible, the threshold is the same as for a steady light, but for flashes of very short duration ($t_0 \ll 0.2$) the visibility depends only upon the total quantity of light received, i.e., the product Ft_0, which must then be equal to $0.21\ F_\infty$.

The aviator flying in clear weather in full moonlight is not in great need of light beacons as the natural illumination is sufficient to make landmarks discernible. For this reason in our experiments we confined the background brightness to an upper limit of full moonlight falling perpendicularly on a white diffusing surface, corresponding to a brightness of 7×10^{-6} candles per square centimeter.

EXPERIMENTS ON THE VISIBILITY OF FLASHING POINT SOURCES

The following arrangement was used. The observer was placed at distances ranging from 1.5 to 6 meters from a vertical white screen 1.5 m $\times$ 1.5 m in size,

[2] These values have been calculated from data given in Russell, Dugan and Stewart's book, Astronomy, Vol. 2, pp. 613, 625, 626, Ginn and Co. (1927). The value of H for starlight is based upon the statement that Rhijn found that the total light of the sky on a clear dark night is 6 times the light from all the stars and that the latter is equivalent to that of 1440 stars of the first magnitude. Thus the brightness H corresponds to that produced by 2160 first magnitude stars at the zenith.

[3] A. Blondel and J. Rey, Jour. de Physique et le Radium **1**, 530–551 (1911).

with its center about the height of the eye. Lamps, behind the observer and screened from his direct view, illuminated the screen so that its brightness could be varied from that of moonlight down to zero. These low brightnesses were measured indirectly by photometering the brightness of a similar surface at one tenth the distance from the lamps and dividing the value by 100.

In the screen there were 150 small round holes ranging from 2 mm^2 to 66 mm^2 area, and one square hole of 1 cm^2 area. The area of the latter could be varied by means of a plane diaphragm so that the opening always remained square and could easily be measured. All the holes were covered on the front by white paper 0.11 mm thick so that they were invisible unless illuminated from behind. When so illuminated by a small lamp, the surface brightness could be varied by either regulating the current or changing the distance of the lamp. Flashing was produced by interrupting the current or by intercepting the light by means of a pendulum shutter. These illuminated spots of light served as point sources.

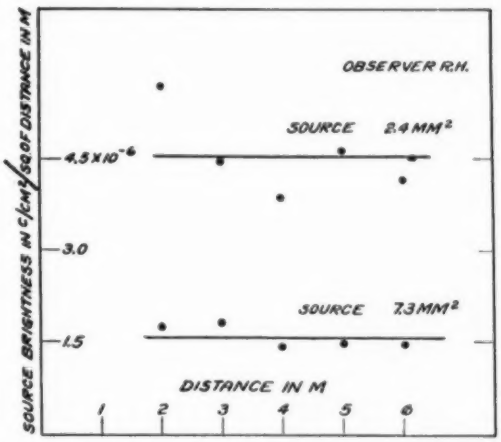

Fig. 1. Visibility of small flashing illuminated areas as affected by their size and their distance from the observer. Screen 1.5 m square with steady illumination of moonlight brightness. The ordinates represent threshold values.

The *threshold value* of the intensity of the point source was determined by gradually decreasing the intensity until the observer lost track of the flashes. This method eliminates all uncertainty as to the location of the light source. In the experiments on threshold intensity, the observer remained in a dark room for a half hour for dark adaptation; then the flashing light was increased until he could easily see it and the observer was asked to signal by tapping at every flash. By reducing the light until his signalling became irregular or stopped we determined the threshold. The values given in Figs. 1 to 7 are the average of three, or more often of ten, observations. This "threshold" will later be used as a unit of light intensity in the determination of the time to locate a flashing light.

If an illuminated area is of sufficiently small size, its visibility will depend upon the total light from it, i.e., upon the product of its area and its surface

brightness. If an area is so large that it is clearly perceived as a surface instead of a point, its effect depends rather upon the contrast in brightness between it and the surrounding surface. Whether an area is recognized as a point or as an extended surface depends upon the visual acuity, and this in turn varies with the brightness of the background illumination. With starlight intensity, or less, a relatively large area is equivalent in its effect to a point source. For an illuminated surface surrounded by a completely black background, Ricco's law states that the total light (area $\times$ brightness) determines the visibility if the angle subtended by the source is less than about one degree. With higher background brightness the total light will determine visibility only for areas which subtend much smaller angles.

It was therefore of interest to determine in our experiments the conditions under which the illuminated areas were effectively point sources. It was also

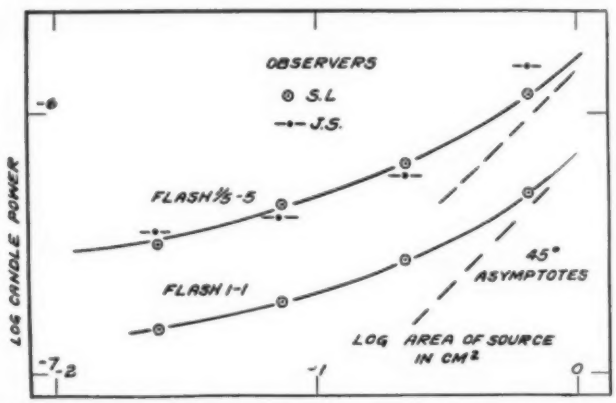

Fig. 2. The effect of the size of the illuminated area and the duration of flashes on visibility. Screen of moonlight brightness at 1.5 m.

desired to know whether the apparent size of the large screen was of importance in determining the threshold intensity of the point sources.

Fig. 1 represents the results of a series of observations in which the brightness of the screen was kept constant at 8×10^{-6} candles per cm^2 and the distance of the observer from the screen was varied from two to six meters, while the square point source was flashed for one-fifth of a second every five seconds. By division of the surface brightness of the source at threshold intensity by the square of the distance to the observer, a value independent of the distance would be obtained, if the source is effectively a point source, and if the screen area is without effect. This appears to be valid within a range of variation that can be accounted for by fatigue of the eye or other personal factors.

It should also be noted that when the area of the source is cut down to one-third the surface brightness for threshold visibility must be increased threefold. A square source of 7.3 mm^2 area at a distance of 2 m, which subtends an angle of 5′ of arc, is small enough to act as a point source with a background of moonlight brightness.

In the experiment, which gave the data shown in Fig. 2, the observer was 1.50 m from the screen having moonlight brightness. The common logarithm of the threshold candle power is plotted against the logarithm of the area of the source. The visibility of small sources depends only on the candle power, but with large areas the threshold candle power increases in proportion to the area, indicating that a constant contrast is required for threshold visibility. From the position of the 45° asymptotes we calculate that the additional brightness to make sources visible that have a subtended angle of more than 0.3 degree is 20 percent for the 1–1 second flashes and 40 percent for the 0.2 second flashes. Areas less than about 3 mm² which subtend an angle of about 5′ are approximately equivalent to point sources. In further experiments the illuminated holes having an area of 2.2 mm² were used as point sources.

The effect of background brightness on the visibility of flashing point sources.

Fig. 3 gives some data on the threshold values of the candle power of flashing point sources as influenced by the brightness of the background illumination. Observations were started after only 10 minutes for dark adaptation; after these the screen brightness was decreased and again four observations taken, etc. Every cross on the graph represents the average of four

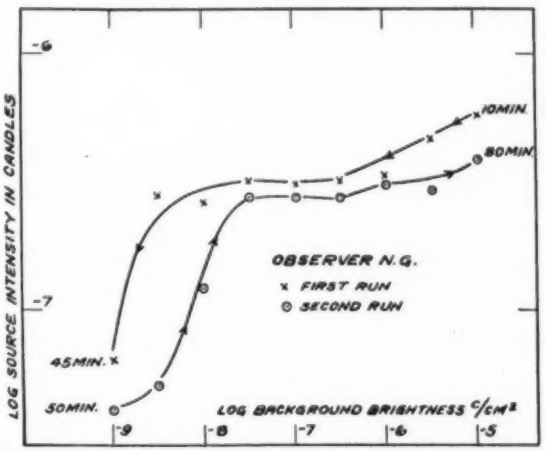

Fig. 3. The effect of background brightness and adaptation on the visibility of flashing point sources. Flashes: 1 sec. on—1 sec. off. Observer 3 m from screen. The arrows indicate the sequence of the observations.

observations. After 50 minutes the screen brightness was increased again and after 80 minutes the original brightness was reached. Apparently a continued dark adaptation stronger than any effects of fatigue must account for the lower values of the second series. Fig. 4 and Fig. 5 show curves for two observers at three and six m distance, and a one second on, one second off flash after one half hour for dark adaptation. The asymptotic values for zero background brightness are indicated by the arrows and are of the same order as those for 10^{-9} candles per cm².

The curves obtained by different observers differ considerably but all agree in showing that the threshold value of candle power increases much

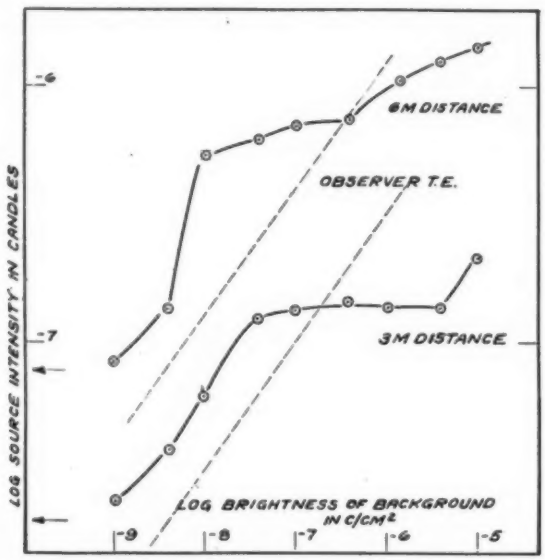

Fig. 4. The effect of background brightness and the distance on the visibility of flashing point sources. Dashed lines correspond to Eq. (5).

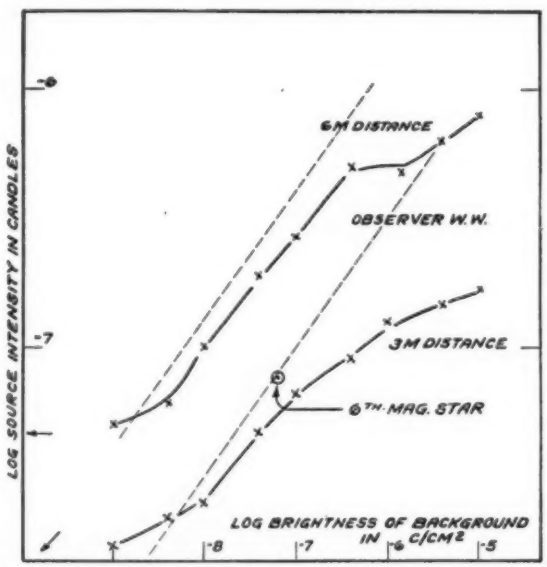

Fig. 5. The effect of background brightness and distance on the visibility of flashing point sources. The dashed lines correspond to Eq. (5).

slower than in proportion to the background brightness. Starlight ($H = 6\times10^{-8}$) is sufficient to require an increase of 5 to 10 fold in the threshold candle power of a point source.

It is interesting to compare these data with astronomical observations on the visibility of stars. Ordinarily a star of the sixth magnitude($F=8.5\times10^{-13}$ lumens/cm^2) is the faintest that is visible to the unaided eye against the background of starlit sky ($H=6\times10^{-8}$). This corresponds to 7.6×10^{-8} candles at a distance of 3 meters. This point has been plotted in a circle in Fig. 5 and is seen to be in fairly good agreement with the observations of point sources at 3 m distance.

Russell[4] finds, however, that stars of magnitude 8.5, for which $F=8.5\times10^{-14}$, are recognizable if the light of the neighboring parts of the sky are screened off. This is in reasonable agreement with the values of F ranging from 1.3 to 2.3×10^{-13} obtained from the threshold candle powers observed with zero background intensity (see arrows in Figs. 4 and 5) by dividing by D^2, D being the distance in cm.

Venus, when of maximum brightness, (magnitude -4.3), can be seen very readily at noon time on a clear day if its exact location in the sky is pointed out, although once lost it may be difficult to find again. Its brightness, which corresponds to $F=1.1\times10^{-8}$ lumens$\times$cm^{-2}, is distinctly above the threshold value of visibility for a point source in a clear sky at noon which has a brightness of the order of $H=1$ candle cm^2.

The following empirical equation seems to be in reasonable agreement with these data.

$$F = C/D^2 = 3.5 \times 10^{-9} H^{1/2} \text{ lumens} \times \text{cm}^{-2} \qquad (5)$$

where F and C are threshold values for a steady point source in a background of brightness H, expressed in candles$\times$cm^{-2}.

TABLE II. *Comparison of observed and calculated threshold values for point sources with backgrounds of various brightness.*

Object	F_{obs}	H	F_{cal}
8.5th mag. star	8.5×10^{-14}	10^{-9} (darkness)	1.1×10^{-13}
6th mag. star	8.5×10^{-13}	5.8×10^{-8} (starlight)	8.5×10^{-13}
4.1 mag. star	—	2×10^{-6} (moonlit sky)	5.0×10^{-12}
1st mag. star	—	5.8×10^{-4} (twilight)	8.5×10^{-11}
Venus (max.)	1.1×10^{-8}	1. (daylight)	3.5×10^{-9}

The second column of Table II gives the light flux density F from various heavenly bodies. The third column gives the background brightness H under which it is observed. The value of F calculated from H by Eq. (5) is given in the last column.

The empirical square root relation seems to give an approximately correct value for the visibility of steady light sources over an extremely wide range of values of H from 10^{-9} to 1.

According to Blondel and Rey's law, as given by Eq. (4), the threshold

[4] H. N. Russell, Astrophys. J. **45**, 60 (1917).

brightness for flashes of 1 second duration is only 21 percent greater than for steady light. Thus the square root relation which we have found for steady sources should be roughly applicable to our experiments with 1 second flashes.

The dotted lines in Figs. 4 and 5 give the threshold candle power calculated by Eq. (5). These straight lines give values of F which agree with the observed within a factor of about 3. The discrepancies appear in large part to be individual characteristics of the observers.

In clear air the visibility of point sources is extremely high. Thus by Eq. (5), or Table II, at a distance of 10 km, a light of 0.11 candles is visible with a completely dark background, 0.85 candles with starlight, 5 candles with moonlight, 85 candles with twilight (stars just visible) and 8500 candles in daylight. These values are to be regarded as lower limits. For signalling purposes we shall see that candle powers at least thirty times greater than these values should be used.

Colored point sources

Similar measurements were made with red, green and violet sources. The color filters used were Wratten No. 70 red-pass beyond 6400A, No. 74 green-pass from 5100–5700A, and No. 76 violet-pass from 3100–4800A. For calibra-

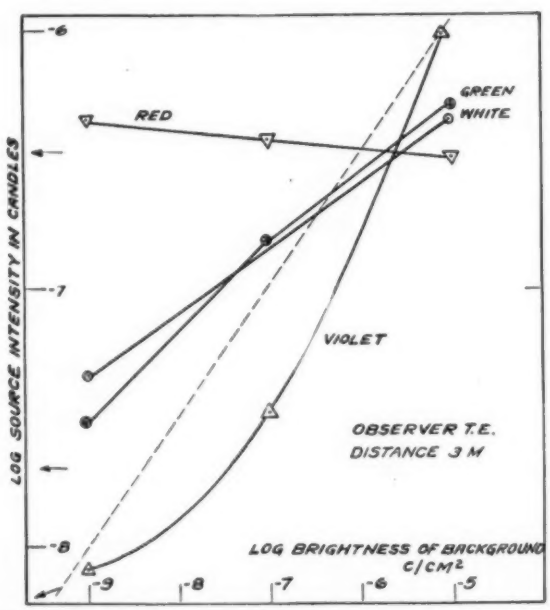

Fig. 6. The visibility of colored flashing point sources as affected by the brightness of a background illuminated by white light. Flashes: 1 sec. on—1 sec. off. Screen at 3 m. Dashed line corresponds to Eq. (5).

tion of the source the colors were matched with white in a photometer with an accuracy of about ± 20 percent. Fig. 6 shows the result of measurements at three m distance with 1–1 second flashes. The visibility of red point sources is independent of the white background brightness; violet point sources are

more than 100 times as visible against an absolutely dark background as against a moonlight background. The high visibility of violet at low intensities is of doubtful practical value for it can be obtained only if the observer does not look straight at the source, the center of the retina being less sensitive for violet. Green light acts like white. The remarkable difference between red and violet is due to the Purkinje effect. It is accounted for by a shift in the visibility curve towards the blue end of the spectrum as the brightness of object and surroundings decreases. Houstoun[5] has plotted relative visibility against the logarithm of the wave-length at high and low brightnesses and finds two curves with a shift of 480A, closely resembling probability curves.

The time to locate a flashing point source of light

In aviation, we are not interested in the intensity at which we lose sight of a beacon, but we do want to know when we can pick it up. Thus as a first requirement we may state that in order to pick up a light, it must have

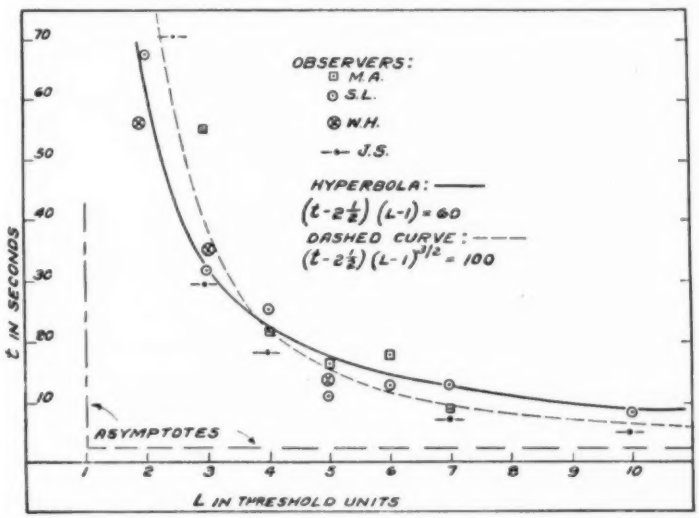

Fig. 7. The time needed to locate a flashing point source in an unknown position within a field subtending a horizontal angle of 56 degrees and a vertical angle of 19 degrees of moonlight brightness, $H=7\times10^{-6}$ c. cm^{-2}. The abscissas give the brightness of the source in threshold units. Flashes: 0.2 sec. every 5 sec.

an intensity above the one determined as threshold in the experiments. If this requirement is fulfilled, we can see the light and next we want to know the length of time necessary to locate it.

To determine this experimentally, we used thirty holes in the screen distributed irregularly over an area of 50 cm height and the full width of 150 cm, leaving 50 cm above and 50 cm below without holes; these blank areas correspond to the sky and the foreground. The flashing light (0.2 second on—4.8 seconds off) could be placed behind any of the holes. This adjustment of the

[5] R. A. Houstoun, Phil. Mag. **10**, 416–432 (1930). See especially p. 420.

flashing source could be done behind the screen unseen by the observer who sat at 1.5 m in front of the screen. The area of each of the holes was 2.24 mm² and the brightness of the screen was moonlight. The measurement itself consisted merely in measuring the time from the instant the flasher circuit was closed and the observer told to look for the light, until the instant he signaled that he had located the flash.

Fig. 7 shows the time as a function of the intensity of the flash expressed in threshold units, each point being an average of ten observations; the use of the threshold unit gives a common scale in spite of different observers having different threshold intensities. An interesting result is that a flash equal to ten times the threshold is seen practically always at the first flash, while it takes three flashes before the light is located if it is only five times the threshold. Naturally the threshold is an asymptote; so is also the 2.5 second line, since the average is half the time interval between flashes if the first flash is seen.

This suggests an hyperbola as an appropriate empirical equation. The full line curve in Fig. 7 is a plot of the equation

$$(t - 0.5T)(L - 1) = 60$$

where T, the period of the flashes, is 5 seconds. A better approximation is obtained by using an exponent $3/2$ for the factor $L-1$. The dashed line curve also plotted in Fig. 7 has the equation

$$(t - 0.5T)(L - 1)^{3/2} = 100. \tag{6}$$

The fact that it takes a relatively long time to find flashing point sources only a few times brighter than threshold proves that they are observed only by a searching process during which the eye scans the whole field in which the point source may occur. Unless the eye happens to be looking at, or the attention is fixed upon, a small area which contains the light at the time of the flash, the flash will not be seen. For flashes of low intensity and short duration, the average finding time should thus be proportional to the period T and to the solid angle ϕ in which the flash is to be sought.

In the foregoing experiments the flashes were located within an area 50×150 cm at a distance of 150 cm, so that the solid angle was $\phi = 0.30$.

We may thus write Eq. (7) in the following form

$$(t - 0.5T)(L - 1)^{3/2} = 67\phi T. \tag{7}$$

This equation which is now generally applicable, gives the average time t required to detect the presence of light flashes of short duration and of period T in an unknown position within a known area which subtends the solid angle ϕ; L measures the light intensity of the flashes in threshold units, that is, in terms of the threshold light intensity of similar flashes (same duration and period) in known positions. The experiments indicate that this equation is valid for background brightnesses ranging from zero up to that of moonlight, the effect of the background being only to change the magnitude of the threshold unit. The equation is probably applicable only when T is greater than one or two seconds.

To obtain a signal which can be detected within 2 or 3 flashes, say within an average of $t = 2.5T$, we see by Eq. (7) that

$$L > 1 + 10\phi^{2/3}$$

Thus with solid angles, such as those within which the aviator must seek beacon signals, the signals should be at least 10 times threshold.

For flashes of one second duration we may safely take as a threshold a value three times that given by Eq. (5) and therefore may conclude that a flashing white light source to be useful as a signal to an aviator should (visible on first or second flash) be of an intensity such that it produces an illumination F at the position of the observer at least as great as

$$F = C/D^2 = 10^{-7}H^{1/2} \text{ lumens} \times \text{cm}^{-2}. \qquad (8)$$

For signals to be transmitted 10 km in clear weather with a background of bright moonlight on a snow covered surface, a point source should then have at least 250 candle power.

The effect of disturbing lights on the visibility of point sources

To find the effect of disturbing lights among which the aviator has to find a flashing beacon, we placed thirty constant lights behind the holes in the screen. The lamps were all connected in series so that simultaneously their

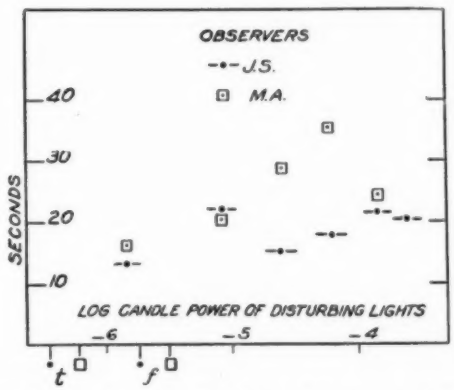

Fig. 8. The time required to locate a flashing point source of 5×threshold intensity in presence of 30 disturbing lights of intensities up to 500× threshold. Flashes every 5 sec.

intensity could be adjusted up to 500 times the threshold of visibility. The flashing light (5 sec. period) was adjusted at five times the threshold for each observer and for each observation placed in an unknown position behind the screen, as in the previous experiment. The finding times were determined with the disturbing lights at different intensities ranging from four times up to five hundred times the threshold. The results were very surprising as can be seen from Fig. 8 where the finding time is plotted against the logarithm of the candle power of the disturbing lights. Apparently the influence of the disturbing lights is very small, the time being less than doubled if the thirty

disturbing lights are made 100 times as strong as the flashing light when this is adjusted at five times the threshold. The marks t at lower margin indicate the thresholds for the two observers, and f the candle power at which the flashing source was adjusted. Every point in the plot represents the average of ten observations.

The disturbing lights in the foregoing experiments were paper-covered holes of 2.24 mm² area illuminated to various brightnesses.

In a later series of experiments, disturbing lights having still greater intensities were produced by illuminating holes having areas 7.3, 22 and 66 mm², the surface brightness in each case being 5×10^{-3} c·cm^{-2}, the candle powers thus being 0.00036, 0.0011 and 0.0033 respectively. The screen was at a distance of 1.5 m.

The observer J. S. as an average of 30 observations with disturbing lights of 0.0033 candles required a finding time of only 12 sec. for a flashing light of 5 times threshold intensity. With lights of 0.0011 candles the time was 11 sec. as an average of 32 observations. Notwithstanding the greater intensities of these disturbing lights the finding times are considerably less than those shown in Fig. 8. It appears that the observer J. S. as a result of continued training, had developed skill in locating the flashing light among the disturbing lights.

The holes in the screen which were used as light sources in these experiments were rather uniformly distributed over its surface. No hole was at a distance of less than 8 cm from its nearest neighbor, so that the flashing light had in no case been within less than 3° from the nearest steady disturbing light. It was thought that this might be the reason for the small influence of the disturbing lights upon the finding time.

Several holes were therefore made at distances of 2 cm from disturbing lights corresponding to angular distances of 0.75 degrees.

Among a set of 60 observations by J. S. were interspersed 28 in which the flashing light was only 2 cm from the nearest disturbing light while in the other 32 the distance was 8 cm or more. In both cases the flashing lights were of 1.7×10^{-6} candles (5 times threshold) while the disturbing lights were 3.6×10^{-4} candles (1100 times threshold). The average finding times were 31 and 6.3 seconds respectively. We may conclude from this experiment that steady lights at an angular distance of three degrees from the beacon have very little effect on its visibility, but that if they are at one degree or less from the beacon they may increase by many fold the time needed to locate it.

Effect of duration and frequency of flashes on visibility of point sources

If our source is a steady light of definite candle power, we may wish to know the best way of interrupting the light to make it most conspicuous. The two variables are: T the flashing period and t_0 the duration of each flash in seconds.

In our measurements we have used as a criterion for conspicuousness the average time to find the flashing light. The apparatus consisted of a small tubular projector with which a star-like spot of light approximately 1 mm²

in area could be focussed on any point of the surface of a white screen 150 cm $\times$150 cm. In front of this projector an interruptor disk was rotated with a speed adjustable between 6 and 240 r.p.m.; the disk was cut out so as to make the light pass during 3/4 or 1/2, 1/4, 1/8, 1/16, 1/32 or 1/64 of the revolution. In front of the disk a shutter was placed which could be opened by pushing a button. Thus at any given instant a series of flashes could be projected on the screen, for which T could be between 0.25 and 10 seconds, and t_0/T between 3/4 and 1/64. The observer sat 2 m in front of the screen with his eyes about 50 cm below the center. At the instant he heard the shutter click he started looking for the light, and with a stop watch the time elapsing between opening the shutter and the observer's signal of locating the light was measured. By taking the average of ten to thirty such observations, we determined the average finding time of a flashing light having a definite T, t_0 and intensity.

The experiments showed that for moonlight, starlight or dark background, and with an interrupted source of fixed candle power, the light is most conspicuous when T lies between 0.5 and 1.0 second and t_0/T is between 0.5 and 0.75. Decreasing T or increasing t_0/T decreases the flashing or flickering appearance and makes recognition difficult. Decreasing t_0/T below 0.5 increases the average finding time (with values of T as low as 1 sec.) because of the lowered visibility of short flashes in accordance with Eq. (4). When T is increased above 1 sec., the finding time is increased because the eye, in wandering over the screen in search of the light, may pass by it during the intervals between flashes, whereas it would not do so with shorter intervals.

When T is greater than one second, there is also another cause which increases the finding time. Even for bright flashes there is a theoretical lower limit for the average finding time which can be verified by experiment. This is determined as follows: the probability of opening the shutter while the light is shut off by the interruptor disk is $1-(t_0/T)$; if the shutter is opened during the dark part of the cycle, the average time elapsing before the light comes on will be $0.5\ (1-t_0/T)T$. For the cases that the shutter opens while the flash is on, the minimum locating time is zero; therefore the average finding time of a flashing light is at least $0.5\ (1-t_0/T)^2T$. For rotating beacons t_0/T is very small, e.g., $5°/360°$ and the minimum average finding time or waiting time is $0.5\times T$, that is, one half of ten. This was checked experimentally, as the finding time for a strong light and T larger than one second is very little different from the theoretical limit.

The relative visibility of flashes of different durations with moonlight background brightness was also determined in connection with these experiments. Results in reasonable agreement with Blondel and Rey's relation, Eq. (4), were found. For example, with a flashing point source in a known location, the threshold intensity for flashes lasting 1 second was only 0.65 of that required for flashes lasting 0.2 second. This result is in reasonable agreement with Blondel and Rey's Eq. (4) according to which the threshold intensity for a 1 second flash should be 0.59 as great as for a flash of 0.2 second.

Rotating beacons

When a rotating beacon is used to produce light flashes, the conditions are in some ways essentially different from those just considered. The lamp used has a definite output in lumens but the beam candles projected toward the aviator varies with the angular horizontal beam width, θ_H, while t_0, the duration of the flash, varies in proportion to $T\theta_H$, T being the period of rotation.

Let us assume that the beacon projects a beam of rectangular cross-section with vertical and horizontal angles θ_V and θ_H, respectively, measured in radians. Then if I is the total luminous output of the beacon in lumens, the candle power within the beam is $I/\theta_V\theta_H$. By Eq. (2) we find that the light flux density within the beam at a distance D from the beacon is

$$F = \frac{I}{D^2\theta_V\theta_H} e^{-D/\lambda}. \tag{9}$$

The duration of the flash is

$$t_0 = (\theta_H/2\pi)T. \tag{10}$$

If the value of F in Eq. (9) is to be equal to the threshold value, we may combine Eqs. (9), (10) and (4) and obtain

$$Ft_0 = F_0(t_0 + 0.21) = (IT/2\pi\theta_V D^2)e^{-D/\lambda}. \tag{11}$$

Since F_0 is constant (independent of t_0), it thus appears that to obtain the most efficient use of the light (minimum I), t_0 and therefore θ_H should be made as small as possible, preferably at least as small as 0.1 sec. With a 10 second period this requires a horizontal beam width of not over 3.6°.

Let us apply these equations to a typical airport beacon of the Department of Commerce design. The concentrated filament lamp is of 1000 watts and gives 22500 lumens. The parabolic mirror subtends an angle of 120° from the lamp filament and thus receives light from a solid angle $\phi = \pi$ or 25 percent of the total. Allowing for intercepted light and imperfect reflection, we may assume 20 percent of the 22500 lumens is utilized in the beam so that $I = 4500$ lumens. Measurements show that the effective beam widths θ_H and θ_V are both 3.5° or 0.061 radians. Thus the average beam candles should be $4500/(0.061)^2 = 1.2 \times 10^6$, which agrees well with experimental determinations with this beacon.

Substituting these values of I, θ_V and θ_H in Eqs. (10) and (11) and eliminating t_0 we find

$$F_0D^2e^{D/\lambda} = \frac{5.6 \times 10^4 T}{1 + 0.046T}. \tag{12}$$

To calculate the maximum distance at which this beacon can be seen, we should put F_0 equal to the threshold value for steady sources, which by Eq. (5) is approximately $F_0 = 10^{-11}$ lumens$\times$cm^{-2} for moonlight background

brightness. Thus if D_1 is the distance at which the intensity corresponds to the threshold value ($L=1$), we have from Eq. (12), after taking the square root

$$D_1 e^{D_1/2\lambda} = 7.5 \times 10^7 [T/(1 + 0.046T)]^{1/2}. \quad (13)$$

If the atmosphere were absolutely clear so that $\lambda = \infty$, the light intensity would vary according to the inverse square law and in this case the beacon could theoretically be seen at a distance of 1.96×10^8 cm or 1960 km, if the period of rotation is $T=10$ seconds or 1.51×10^8 cm (1510 km) if $T=5$ seconds.

The absurdity of this result indicates that the maximum range of such bright light sources at night is always determined by the scattering or absorption coefficient, so that it is not justifiable to put $\lambda = \infty$. The distance in which the intensity of a parallel beam falls to 1/10th because of scattering or absorption is 2.3λ and this may be roughly taken to be equal to the distance denoted by the term "visibility" in aeronautical weather reports.

The columns headed D_1 in Table III give values of the maximum range D_1 for a series of different visibilities and for three rates of rotation of the beacon corresponding to periods of 5, 10 and 20 seconds respectively. These data have been calculated from Eq. (13) by plotting curves giving λ as a function of D_1 (for each of the three values of T).

It should be noted that with more rapid rotation the amount of light received in each flash decreases. If the visibility were really unlimited ($\lambda = \infty$), the ranges would be in the ratio 1:(1.30):(1.59) for $T=5$, 10 and 20 sec. respectively.

Examination of the data of Table III shows that the effect of the limited visibility is to bring these ratios far closer to unity. For example, with $\lambda = 10^5$ (1 km), the ratios are 1:(1.05):(1.09). Thus the decreased flash intensity caused by rapid rotation becomes of minor importance even in determining the maximum range.

TABLE III. *Pick-up of department of commerce type of rotating beacon by approaching airplane moving with velocity $v=5000$ cm sec. ($=180$ km/hr. $=112$ miles/hr.); $\phi=0.5$; $T=$ period of rotation in seconds.*

λ	Visibility 2.3λ	Max. range D_1 (cm)			Additional distance for pick-up Δ (cm)			Effective range $D_1-\Delta$ (cm)		
		$T=5$	10	20	5	10	20	5	10	20
10^7 cm	230.0 km	$3.18 \cdot 10^7$	$3.49 \cdot 10^7$	$3.76 \cdot 10^7$	$3.49 \cdot 10^8$	$4.64 \cdot 10^8$	$5.96 \cdot 10^8$	$2.83 \cdot 10^7$	$3.03 \cdot 10^7$	$3.16 \cdot 10^7$
10^6	23.0	$6.35 \cdot 10^6$	$6.78 \cdot 10^6$	$7.10 \cdot 10^6$	$8.8 \cdot 10^5$	$1.12 \cdot 10^6$	$1.37 \cdot 10^6$	$5.47 \cdot 10^6$	$5.66 \cdot 10^6$	$5.73 \cdot 10^6$
10^5	2.3	$1.00 \cdot 10^6$	$1.05 \cdot 10^6$	$1.09 \cdot 10^6$	$1.83 \cdot 10^5$	$2.24 \cdot 10^5$	$2.70 \cdot 10^5$	$8.2 \cdot 10^5$	$8.3 \cdot 10^5$	$8.2 \cdot 10^5$
5×10^4	1.15	$5.60 \cdot 10^5$	$5.83 \cdot 10^5$	$6.00 \cdot 10^5$	$1.12 \cdot 10^5$	$1.36 \cdot 10^5$	$1.68 \cdot 10^5$	$4.48 \cdot 10^5$	$4.47 \cdot 10^5$	$4.32 \cdot 10^5$
2×10^4	0.46	$2.54 \cdot 10^5$	$2.66 \cdot 10^5$	$2.71 \cdot 10^5$	$5.8 \cdot 10^4$	$7.40 \cdot 10^4$	$1.00 \cdot 10^5$	$1.96 \cdot 10^5$	$1.92 \cdot 10^5$	$1.71 \cdot 10^5$
10^4	0.23	$1.39 \cdot 10^5$	$1.45 \cdot 10^5$	$1.48 \cdot 10^5$	$3.72 \cdot 10^4$	$5.03 \cdot 10^4$	$7.58 \cdot 10^4$	$1.02 \cdot 10^5$	$9.5 \cdot 10^4$	$7.2 \cdot 10^4$
5×10^3	0.12	$7.6 \cdot 10^4$	$8.0 \cdot 10^4$	$8.1 \cdot 10^4$	$2.52 \cdot 10^4$	$3.80 \cdot 10^4$	$6.29 \cdot 10^4$	$5.1 \cdot 10^4$	$4.2 \cdot 10^4$	$1.8 \cdot 10^4$

Pick-up of beacon by an approaching airplane

We have seen in the derivation of Eq. (7) that a considerable time interval is necessary to "pick-up" a flashing source of light if its visibility is only little above the threshhold value. A rapidly moving airplane may thus approach within a distance D, which is far less than the maximum range D_1, be-

fore the signal will be seen. To estimate the effective range of a beacon we want to determine this average distance D.

If, as before, L denotes the intensity of the beacon flashes in threshold units, at any distance D, we have, by Eq. (11)

$$L = (D_1/D)^2 e^{(D_1-D)/\lambda}. \tag{14}$$

Consider an airplane moving directly towards a beacon. When it reaches a point corresponding to D_1, the value of L is unity. Let x measure the distance along the path of the plane from this point so that $x = D_1 - D$. Taking the logarithm of Eq. (14) and differentiating at the point $x = 0$, we have

$$\frac{d \ln L}{dx} = \frac{1}{\lambda} + \frac{2}{D_1}.$$

Thus for reasonably small values of x the variation of L with x is given by

$$L = \exp\left[x\left(\frac{1}{\lambda} + \frac{2}{D_1}\right)\right]$$

or which may be written

$$L = e^{x/\lambda'} \tag{15}$$

where

$$\frac{1}{\lambda'} = \frac{1}{\lambda} + \frac{2}{D_1}. \tag{16}$$

Inspection of Table III shows that D_1 is considerably larger than λ. The variation of L due to absorption is thus usually much greater than that due to the inverse square law at distances such that the beacon is barely visible.

Eq. (7) gives us the average time t required to pick up a flashing light having flashes of uniform intensity. In the present case the successive flashes are of increasing intensity. The reciprocal of t is the probability per second of picking up the light. The average distance Δ which the observer must travel towards the beacon before seeing it is thus the distance over which the time integral of this probability is unity. Thus

$$\int_0^{\Delta} \frac{dx}{vt} = 1$$

where v is the velocity of the plane towards the beacon. Substituting into this the value of t given by Eq. (7) and the value of L from Eq. (15) and Eq. (16) we obtain

$$\phi T v/\lambda' = \int_0^{\Delta/\lambda'} \frac{du}{\frac{1}{2\phi} + 67(e^u - 1)^{-3/2}}. \tag{17}$$

When Δ/λ' is sufficiently small the term $1/(2\phi)$ can be neglected and we have

$$\left.\begin{aligned} 67\phi Tv/\lambda' &= (\tfrac{2}{3})p^{3/2} - 2p^{1/2} + 2\tan^{-1}p^{1/2} \\ \text{where} \qquad p &= e^{\Delta/\lambda'} - 1 \end{aligned}\right\} \tag{18}$$

When $\Delta/\lambda' < 0.1$, this reduces to

$$67\phi Tv/\lambda' = (2/5)(\Delta/\lambda')^{5/2}. \tag{19}$$

For large values of Δ/λ', $(e^u - 1)^{-3/2}$ may be replaced by $e^{-3u/2}$ and integration then gives

$$Tv/\lambda' = 2[(\Delta/\lambda') + (\tfrac{2}{3})\ln(1 + 134\phi e^{-3\Delta/2\lambda'}) - K] \tag{20}$$

where K is an integration constant.

In applying these results to the beacon we are considering, let us place $\phi = 0.5$. This corresponds to a field of view, in which the beacon is to be sought, measuring 20° vertically and 82° horizontally. With this value of ϕ, Eq. (18) is found to be reasonably accurate for all values of $\Delta/\lambda' < 1$ and Eq. (20) which holds accurately for $\Delta/\lambda' > 3$ reduces to

$$\phi Tv/\lambda' = \Delta/\lambda' - 2.897 + 0.285\log_{10}(1 + 67e^{-3\Delta/2\lambda'}).$$

In the range from 1 to 3 numerical integration was resorted to.

Table IV gives some typical values of Δ/λ' and $\phi Tv/\lambda'$ from which a curve may be prepared.

TABLE IV. *Table of values of the integral of Eq. (17) for the case $\phi = 0.5$. Below about $\Delta/\lambda' = 1$ these values are independent of ϕ.*

Δ/λ'	$\phi Tv/\lambda'$	Δ/λ'	$\phi Tv/\lambda'$	Δ/λ'	$\phi Tv/\lambda'$
0.1	1.98×10^{-5}	1.4	0.0310	5.0	2.124
0.2	1.21×10^{-4}	1.8	0.0737	6.0	3.10
0.4	7.75×10^{-4}	2.2	0.1516	8.0	5.10
0.6	2.36×10^{-3}	2.6	0.281	10.0	7.10
0.8	5.25×10^{-3}	3.0	0.470	20.0	17.10
1.0	1.070×10^{-2}	4.0	1.204		

For $\Delta/\lambda' > 6$; $\phi Tv/\lambda' = \Delta/\lambda' - 2.90$

In Table III are given values of Δ calculated for a plane moving towards the beacon with a velocity of 5000 cm sec.$^{-1}$ (180 km per hr. or 112 miles per hr.). This represents the average additional distance the plane must travel from the point where threshold visibility is reached before the beacon is seen within a large field which subtends a solid angle $\phi = 0.5$. These values of Δ were calculated by means of a curve prepared from the data of Table IV. The values of λ' were first calculated by Eq. (16) from λ and D_1 and then $\phi Tv/\lambda'$ was obtained; the corresponding value of Δ/λ' from the curve multiplied by λ' then gave Δ.

It should be noted that Δ is relatively large. For example, with a "visibility" of 23 km and with a beacon rotating once in 10 seconds the plane will

travel on the average a distance of 11.2 km from the point where the threshold is reached before the beacon is seen. It takes the plane 224 seconds to travel this distance so that an average of 22.4 flashes above threshold value is needed before the beacon is observed. At the end of this time the intensity of the light from the beacon is 4.25 threshold units. The effective range, which is perhaps the best measure of the value of a beacon under given conditions of visibility, is thus $D_1-\Delta$ as given in the last three columns of Table III. It should be observed that with the highest visibility, slightly greater effective ranges are obtained with $T=20$ sec. than with 10 or 5 seconds. But with a "visibility" of 2.3 km the range is greater for $T=10$ sec. than for either 5 or 20. With still poorer visibility the maximum range shifts gradually to the beacon with $T=5$. Similar calculations show that with λ as low as 10^4 the maximum effective range occurs with T about 2 seconds.

These results indicate that under conditions of poor visibility considerably more rapid rates of rotation than the standard 10 second period are desirable. The lowering of the light effect of each flash, which causes D_1 to decrease with T, is more than offset by the greater ease and rapidity with which the more frequent flashes can be detected in unknown positions within the large field of view presented to the aviator.

Visibility of Flashes of Diffuse Light

The light from cities reflected from the lower surfaces of high stratus or cirrus clouds can frequently be seen from the ground at distances as great as fifty miles on clear dark nights. Similarly an aviator flying high above clouds covering a city can see an illuminated area in the upper surface of the clouds at great distances. In these cases recognition of the source of light is facilitated by the contrast in intensity between the illumintated area and its surroundings.

If the aviator flies at night within a cloud over a city he may have difficulty recognizing the increased intensity of diffused light because of its uniformity and steadiness. Charles A. Lindbergh has stated to one of us that under such conditions the flash of green light produced by a trolley was visible when large steady flares at a nearby airport were not recognizable.

Useful signals can thus be transmitted through fog by sudden flashes of a light source which is producing a general illumination of the fog.

A rotating beacon immersed in a fog or just below low clouds produces a nearly steady or only gradually changing illumination of the fog. Definite interruptions by means of a shutter mechanism may increase the visibility under these conditions and the best results could then be obtained if the beacon were pointed toward the zenith.

With a desire to overcome the loss of efficiency due to interrupting a steady source and to obtain flashes of short duration and great intensity, the device illustrated in Fig. 9 was constructed. The flash is produced by making and breaking a contact between an iron rod cathode and a copper block as anode. A large reactor stores up energy during a short circuit period and delivers it to the arc when the electrodes are separated. This increases the dura-

tion and intensity of the arc and reduces the maximum current and voltage required. The device is fed by d.c. line or from a 9 kw 150 V d.c. generator or mercury rectifier outfit.

The maximum arc current is about 200 amps. The surge resembles a half cycle of sinusoidal wave and is of a total duration of about 0.2 sec. The device is usually adjusted to give a flash every two seconds. A reflector is used to throw the light which would strike the ground up into the sky. The light output per flash is about 50,000 lumen seconds, and the effective duration is about 0.1 second, and thus according to Eq. (4) the visibility of these flashes should be the same as those of an interrupted steady source of $F_\infty = 160000$ lumens.

Fig. 9. Flashing arc beacon.

The range of visibility of signals of this kind depends primarily on the ability of the observer to distinguish the diffuse flash from the steady background illumination. Many experimental investigations have been made of the minimum change of brightness required to produce a perceptible effect. The results are summarized in Weber's law that the smallest detectable change is a constant fraction of the total illumination. Most observers find that a 2 percent change in brightness is observable. These measurements, however, have practically all been made with far higher background intensities than those that interest us, and the brightness of only part of the field was changed, leaving the remainder steady.

In order to study this problem from the point of view of the aviator, Mr. C. C. Steffens has conducted a series of measurements in this laboratory. His results are summarized in Figs. 10 and 11.

A white sheet 2 meters square was hung up in a dark room 1.8 m from the observer. The sheet was uniformly illuminated by 4 tungsten lamps placed behind the observer opposite the 4 quarters of the screen. Each lamp

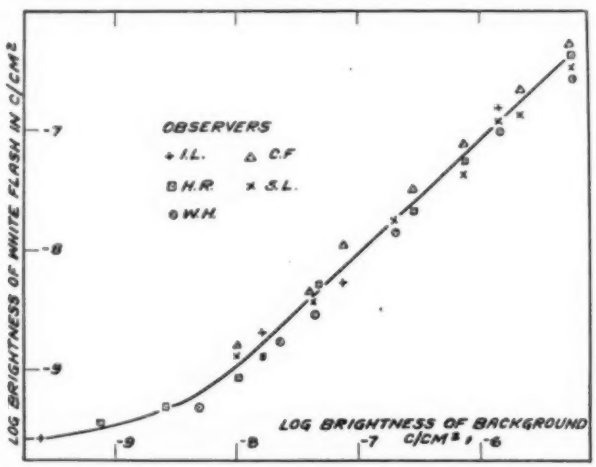

Fig. 10. The effect of steady background brightness on the visibility of diffuse flashes of white light. The ordinates represent the intensities needed for the detection of one-half the flashes.

was mounted inside of a small box which was open at one end and which contained two ground glass diffusing screens between which diaphragms of vari-

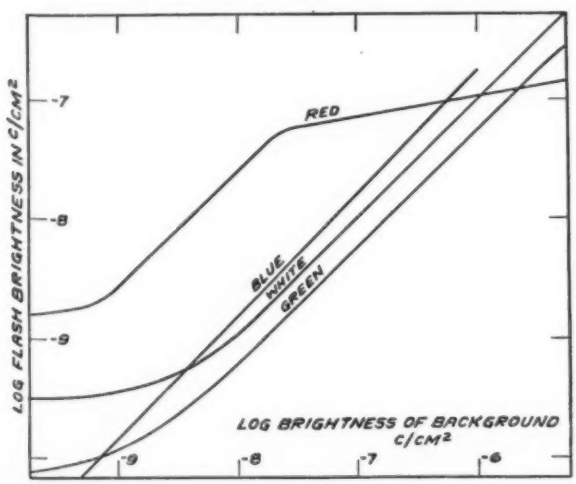

Fig. 11. Visibility of diffuse flashes of colored light as affected by the brightness of steady illumination by white light. The ordinates are similar to those of Fig. 10.

ous openings could be interposed. The intensity of the screen illumination was varied by changing the current through the lamp filaments, but the lamp was never operated at below one-tenth normal candle power in order

not to change the color too greatly. Greater candle power variations than 10:1 were obtained by using the diaphragms whose diameters ranged from 4 down to 0.03 cm. The lamps and larger diaphragms were calibrated by photometering the screen with reasonably high brightness obtained by placing the lamps at suitable distances from the screen. Tests showed that the variation of color by the 10:1 change in filament brightness did not cause appreciable effects for the same results were obtained by changing the brightness by means of the diaphragms.

Flashes of light were superimposed upon the steady screen illumination by turning on and off lamps in four other similar boxes. Shutters were avoided for the sound produced by them could lead to psychological errors in detecting the flash. The currents were actually turned on and off silently by means of a thyratron circuit in which the time interval could be adjusted.

After the observer had reamined in the dark room long enough for dark adaptation, the steady screen brightness was adjusted to any desired value from zero up to that of full moonlight (7×10^{-6} c/cm^2).

Flashes were then made at irregular intervals starting at intensities high enough to be easily seen, decreasing rapidly until no flashes were seen and then increasing the intensity slowly until about fifty percent of the flashes were seen. This kind of effective threshold proved to be surprisingly definite and reproducible.

Fig. 10 gives data obtained by five observers with 1-second flashes of white light. The full line on the logarithmic plot which best represents the data is practically straight and of a slope 0.95 in the range of background brightnesses between 10^{-8} and 7×10^{-6} c/cm^2. Below 10^{-8} the threshold flash rapidly becomes a larger fraction of the background brightness until with a background of 2.7×10^{-10} the flash must be equal to the background in intensity.

The relation of the intensity of the flash to the background is thus given in Table V. At brightnesses ranging from 10^{-4} to 1 Weber's law holds and the ratio remains constant at about 0.02.

TABLE V. *Relation between brightness of a 1-second flash of white light which is 50 percent visible, and the white background brightness.*

Background	Flash: Background
2.7×10^{-10} c·cm^{-2}	1.00
10^{-9}	0.24
10^{-8}	0.112
10^{-7}	0.095
10^{-6}	0.080
$7\cdot10^{-6}$	0.073

These flash intensities are such that on the average one-half are noted by the observer. With flashes close to these threshold values the observer often expresses doubt as to whether a flash that is noted is real, but actually, with *white flashes*, it is an extremely rare occurrence for the observer to report a flash if one did not actually occur. On the other hand, with threshold flashes of *red light* 30 or even 40 percent of the reported flashes were imaginary.

When the intensity of the flashes is made twice that of the threshold, the flashes are unmistakable, even though the observer is not particularly attentive. Every flash is seen under these conditions.

With a background intensity as high as moonlight (7×10^{-6}), a 1-second threshold flash is of such duration that its beginning and end are separately recognizable. With background brightnesses greater than 1.6×10^{-8} (0.4 of starlight) the flash brightness needed for threshold visibility is the same for 1 second and for 5 second flashes. With a background brightness less than about 6×10^{-9} (0.1 starlight), threshold flashes produce the sensation that the light gradually increases in brightness and slowly fades away. With a background at 2.7×10^{-10} c·cm^{-2}, a 1-second threshold flash is seen only after a period of from 2 to 4 seconds after its beginning and the flash seems to persist for 6–10 seconds after it has really ended. With a flash of twice threshold intensity, both of these time lags drop to about 1 second.

With these low background intensities (less than 0.1 starlight) it was observed that a 5-second flash only needed to be 0.5 as bright as a 1-second flash for 50 percent visibility. This seems to indicate that at these low intensities the numerical constant in the Blondel-Rey Eq. (4) should be 1.7 seconds in stead of 0.21. At intensities above starlight, however, we are probably justified in using the unchanged Eq. (4) to calculate the visibility of flashes of very short duration. Thus the candle-sec·cm^{-2} required for a theshold *instantaneous flash* may be obtained by multiplying the flash brightness, as given by Fig. 10 or Table V, by 0.17 seconds (i.e., $0.21\div1.21$). For very low intensities, however, this factor should be 0.63 (i.e., $1.7\div2.7$).

Very interesting changes in the appearance of the illuminated screen were observed as its brightness was gradually decreased. At brightnesses from 1 to 3×10^{-8} c·cm^{-2}, the screen appeared to be remarkably uniform in intensity and perfectly steady, and seemed to be of a gray-green color resembling the appearance of a window opening out onto a vast open space on a starlit night. At intensities above 3×10^{-8} the resemblance to an open window disappeared and it appeared to be a *screen*.

As the intensity was decreased a remarkable change in quality occurred quite critically, usually at about 8×10^{-9} c·cm^{-2}. The appearance of the open window vanished and was replaced by an area of very nonuniform intensity of yellowish color. If the eyes were directed at the center of the screen, the central part appeared to be darker than the edges. The central part showed fluctuations of intensity both in time and space while the edges appeared steady. The edges even seemed to be brighter than when the actual brightness was greater. Thus as the light intensity was deliberately decreased through the critical regions, the center of the screen appeared to grow darker, but the edges seemed to grow brighter. The fluctuations became slower and of larger size as the intensity decreased in the range below the critical value. Undoubtedly these effects result from fluctuations in the number of quanta of light striking the cones in the central part of the retina. At a background brightness of 3×10^{-10} c·cm^{-2}, areas subtending 5° to 10° appeared to increase rapidly in brightness and gradually (in about 5 seconds) fade away. The ap-

parent size seemed to correspond roughly with that of the smallest black object that could be recognized against the black screen (visual acuity).

Since the aviator will rarely have to deal with background intensities less than 0.1 starlight (6×10^{-9} c·cm^{-2}), effects of these kinds will not often be observed.

In connection with these experiments a study was made of the ability of the observer to detect gradual changes in light intensity such as might occur when an aviator flies at night in fog towards a city. Slow steady changes in brightness increasing or decreasing at about 3 percent per second were produced beginning at unknown times, and the observer was asked to report any change, and also to describe the brightness in terms of the original brightness.

With moonlight background an increase was first noticed when the brightness had increased an average of 45 percent. With 0.1 moonlight 33

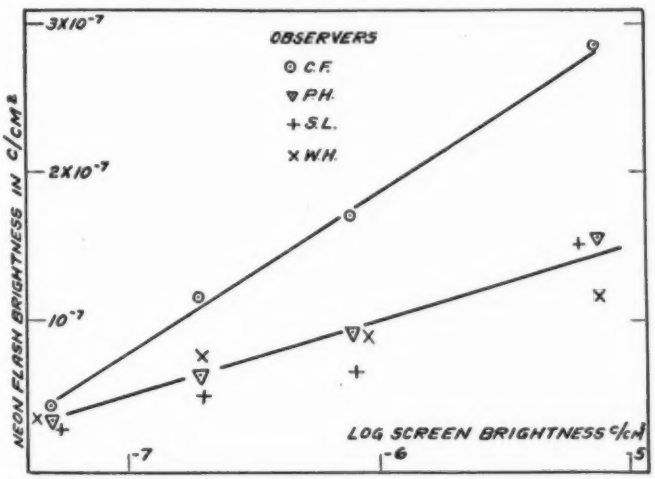

Fig. 12. The visibility of flashes of light from a neon arc as affected by white background brightness.

percent is seen, while with starlight a 20 percent change is needed. The results were, however, extremely irregular. For example, in some cases an increase in brightness was not noted until the brightness had changed in the ratio 1:2. Often an increase or decrease was reported to continue for a considerable time after it had stopped. Estimates of absolute intensities were approximately proportional to the three-halves power of the true intensities with an average error of ± 40 percent.

Roughly, therefore, we can conclude that gradual changes in light intensity are not noticeable with certainty until the intensity has changed in the ratio 1:2 or even 1:3.

Studies were also made of the visibility of *colored flashes*. These data are summarized in Fig. 11 on which for comparison the curve for white light from Fig. 11 is reproduced. The red light examined by a spectroscope showed a band lying mostly between 5900–6900. The green lamp gave a band mostly

between 5000–5700, while the blue gave 4500–4900. The lamps were photometered only at high intensities to avoid the Purkinje effect.

The data of Fig. 12 were obtained with a neon lamp. The lower curve agrees well with the "red" curve of Fig. 11.

With red flashes the threshold value remains nearly constant at about 10^{-7} c·cm^{-2}, while the white background brightness decreases from moonlight down to about 0.5 starlight (2×10^{-8} c·cm^{-2}). In this range the threshold flash was observed as a change in color but not as a change in intensity of the light on the screen. With background intensities less than this, the brightness of a 1-second red flash needs to be about 30 times that of the white background and the flash then does not produce any appreciable change in color of the background, but appears as a change in its brightness.

Flashes of diffuse red light are thus far less advantageous for the transmission of visible signals than are flashes of white or green. At high background intensities, such as moonlight, however, less red light than white light would be needed.

We may use the foregoing data to calculate the visibility of the flashes produced on a white diffusing surface, such as that of a cloud, by the light from a flashing point source at a distance D from the surface.

TABLE VI. *The candle power of a point source needed to give a distinctly visible 1-second flash (2×threshold) on a white diffusing surface (cloud or snow) at distance D. H = steady illumination of the surface in c·cm^{-2}.*

H	$D=1$ km	$D=2$ km	$D=5$ km
10^{-9}	15 c	60	375
10^{-8}	71	284	1770
10^{-7}	600	2400	15000
10^{-6}	5000	20000	125000
7×10^{-6}	32000	128000	800000

The data given in Table VI have been calculated by means of Eqs. (1) and (3) using the factors in Table V, but multiplying them by 2 in order to have flashes that can be seen with certainty. It has been assumed that the light from the source falls at normal incidence on the white surface which reflects 100 percent of the light according to Lambert's cosine law. If we have a surface which reflects the fraction R and if the incident light beam makes an angle θ with the normal to the reflecting surface, then the candle powers in Table VI are to be divided by $R\cos\theta$.

On a dark cloudy night when $H=10^{-8}$ (0.17 starlight), it is thus possible to see the light flashes reflected down from clouds at a height of 1000 m from a flashing point source of only 71 candles on the ground. But if the cloud surface were illuminated by full moonlight, it would be necessary to increase the point source from 71 to 32,000 candles to produce an equally visible flash at the same distance.

With flashes of very short duration, we have seen that the candle seconds needed are only about 1/6th of the candles of the steady light needed to produce 1 second flashes. The rotating beacon which we have considered gives

1.2×10^{6} beam candles for 0.1 second in each flash and thus by Table VI visible diffuse flashes should be observed when its beam at a distance of 4.7 km from the beacon falls at normal incidence on a cloud illuminated by full moonlight. On a dark night, with absolutely clear air and $H=10^{-8}$, such flashes should be visible at 100 km.

The flashing arc beacon (Fig. 9) giving 50,000 lumen-seconds (or 8000 candle-seconds in upper hemisphere) should thus produce visible flashes at 1.25 km on clouds exposed to bright moonlight or at 26 km when the steady light on the clouds is 10^{-8}.

Comparative visibility of diffuse light and light from point sources

In column 2 of Table VII are given, for a series of steady background brightnesses, the values of light flux density F from a flashing point source needed for threshold visibility, as calculated by Eq. (5). For comparison with these data we have, in column 3, the values of F which are visible according to the data of Table V when rendered diffuse by falling on a perfectly white diffuse reflector. Here we have made use of the relation between F and H given by Eq. (3).

TABLE VII. *Comparative visibility of diffuse light and light from point sources, both giving one second flashes.*

Background brightness H c·cm^{-2}	Light flux density F for threshold visibility of point source	Light flux density F for threshold visibility after diffusion from white surface	Ratio of sensitivity	
			Point source vs. diffuse source	Photocell vs. visual for diffuse light
10^{-9}	1.1×10^{-13}	7.5×10^{-10}	6800	6
10^{-8}	3.5×10^{-13}	3.5×10^{-9}	10000	28
10^{-7}	1.1×10^{-12}	3.0×10^{-8}	27000	240
10^{-6}	3.5×10^{-12}	2.5×10^{-7}	72000	2000
7×10^{-6}	9.2×10^{-12}	1.6×10^{-6}	170000	13000

It is seen that the power of the eye to focus the light from a point source upon a small part of the retina makes it from 7000 to 170,000 times more sensitive to light from point sources than to diffuse light which cannot be so concentrated. This advantage increases roughly in proportion to the square root of the background brightness, probably largely as a result of the increasing acuity of vision.

In considering the values of the ratios given in the fourth column of Table VII, it should be recognized that the time needed to find a flashing point source is an important factor, while with flashing diffuse lights the average time needed to detect the signals will not exceed one-half of the period of flashing. Thus, as we have seen, the flashing point sources for practical reasons need to be 10 or 20 times the threshold, while a factor of 2 is sufficient to make diffuse flashes immediately visible.

The photoelectric cell as a detector of diffuse modulated light.

Although the human eye is extremely sensitive to the light from point sources, it becomes many thousands of times less sensitive when the light is

diffused, as for example by fog. The photoelectric cell on the other hand gives a response which is determined by the total light that falls upon it, and therefore it should possess particular advantages as compared to the eye for detecting diffuse light.

We have seen how the flashing of diffuse light signals for visual observation gives them the characteristics needed to distinguish them from the steady diffuse light such as moonlight or the light from cities. In using the photoelectric cell, however, it is better to modulate the light at a definite frequency, such as 1000 per second, and thus be able to distinguish the signal light from all others by using a tuned amplified for the photo-cell current.

In order to determine how well such a system may be used to detect diffuse light we have constructed a special photocell and amplifier equipment shown diagrammatically in Fig. 13.

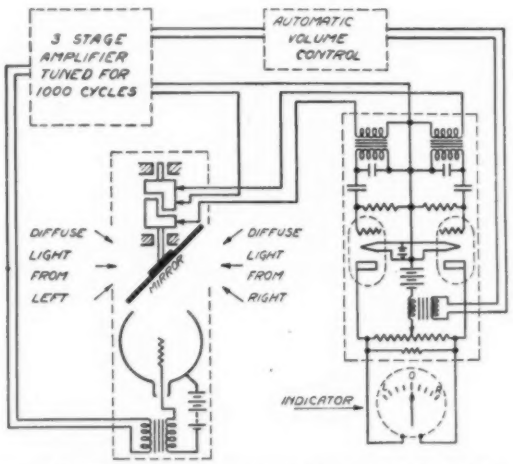

Fig. 13. Photoelectric receiver for modulated light.

The photoelectric cell should be of large light gathering power. We used a PJ-19 argon-filled caesium photocell which has been developed for television outfits. The spherical bulb was 19 cm diameter and had a window opening 15 cm in diameter or 175 cm² area. It was operated with 45 volts on the anode and gave a sensitivity of 10 microamperes per lumen for white light and for the light from a mercury arc and about 8 microamperes per lumen for light from a neon lamp.

In front of the window of the photocell was a mirror which could be rotated slowly (200 to 300 r.p.m.) about an axis inclined at 45° to the mirror surface. In this way the diffuse light from the right and left sides (of the airplane) could be thrown alternately and periodically into the photocell.

The alternating current component from the photocell passed to an amplifier consisting of 3 stages of screen-grid amplification (UY224 tubes), the first two stages being tuned for 1000 cycles. The amplifier was thus highly selective for the modulated light signals. The voltage amplification factor was about 2,000,000.

On the shaft of the rotating mirror was a commutator through which output of the amplifier was passed. This acted as a single-pole double-throw switch which alternately threw the amplified 1000-cycle current to two detector tubes (UX 112A) in push-pull arrangement as shown at the right hand side of Fig. 13.

In this way the modulated light that entered the photocell from the *right* tended to produce a deflection to the *right* of the needle of the d.c. indicating instrument which had a sensitiveness of 150 microamperes for full scale.

The indicator reading thus gave a measure of the *difference* in the diffuse light intensities on the two sides of the photocell. The frequency of rotation of the mirror was immaterial so long as it was more rapid than the indicator needle could follow.

This differential arrangement was adopted so that the direction of the light source could be determined even in presence of fog.

When desired, the light intensity in either direction could be determined by properly fixing the position of the mirror after stopping its rotation, or more conveniently by rendering one or the other of the detecting circuits inoperative.

When using the differential arrangement it is usually desirable to have an indication which measures the relative rather than the absolute difference in the intensities in the two directions.

This was accomplished by utilizing the automatic volume control illustrated in Fig. 13. In this device the 1000-cycle component of the output of the detector tube circuit was rectified by a UX 112A tube and this rectified voltage was used to bias the grid of the first tube in the amplifier. A filter circuit eliminated the fluctuations caused by the mirror rotation so that the *average of the intensities* of the amplified 1000-cycle current produced by the light from the two sides was kept nearly constant as the actual light intensity increased. The *difference in the intensities* from the two sides was, however, given by the indicating instrument.

To test the sensitivity of this apparatus a source of modulated light of known intensity was constructed by using a shutter operated by a synchronous motor. The light intensity was made to increase and decrease linearly with time as shown by diagram *c* in Fig. 14, with a frequency of 1000 per second. The intensity varied from zero at the minimum to 0.5 candles at the maximum so that the average candle power was 0.25. The amplitude A_1 of the 1000-cycle component in a Fourier series expansion, $I = A_1 \sin \omega t$, corresponds to 0.2 candle (i.e., 80 percent of the average 0.25).

Two large sheets of white cardboard were mounted in vertical planes at right angles to one another and placed at a distance of 40 meters from the above described source of modulated light so that the light fell on each plane at an angle of incidence of 45°. The brightness of illumination of these sheets was thus 2.8×10^{-9} candles cm^{-2} (i.e., $0.2 \times 0.71/\pi(4000)^2$) in terms of the amplitude of the 1000 cycle component.

The photocell outfit was mounted symmetrically between the sheets so

that when the mirror was rotated the cell was exposed alternately to the diffused light from the two sheets.

By making a slight adjustment in the inclination of one of the sheets, with respect to the light rays from the source, the needle of the indicator instrument was brought to zero (see Fig. 13). A change of the inclination of either sheet by as little as 0.9° then produced a distinct deflection of the needle (about 15 microamperes). This corresponds to a change in brightness of 1.5 percent of the average brightness (2.8×10^{-9}). Thus the photocell amplifier outfit was sensitive to a change in the absolute brightness of only 4×10^{-11} candle$\times$cm^{-2}, which is only about $\frac{1}{6}$ of the threshold of the eye for flashing lights with a completely dark background (see the data of Fig. 10).

A particular advantage of the photocell equipment is its complete insensitiveness to the steady background illumination. Thus the sensitiveness to modulated light was not appreciably altered by letting light from in-

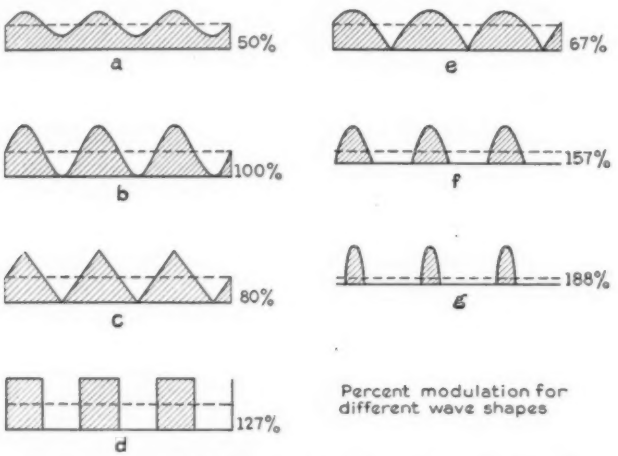

Fig. 14. The effectiveness of various types of modulation.

candescent lamps fall on the sheets up to a brightness of 10^{-3} candle$\times$cm^2 (about 100 times moonlight). With full daylight or sunlight, however, the photocell was overloaded and the sensitiveness to modulated light decreased greatly. Undoubtedly special cells and circuits can be devised which would make it possible to detect very weak signals of diffuse modulated light even in presence of sunlight.

It is interesting to compare this observed sensitiveness of the photocell apparatus with the corresponding visual data given in Tables VI and VII.

As before we may assume that a reliable signal must produce a brightness of twice threshold value. Thus for the photoelectric detector we need a light flux density $F=2.5\times10^{-10}$ lumens$\times$cm^{-2}. This corresponds to 2.5 candles at 1 km. The last column in Table VII gives a factor which measures the relative advantage of the photocell detector over visual observation for diffuse light signals. This factor is obtained by dividing the values of F in the third column by the threshold value for the photoelectric detector (1.26×10^{-10}). In full

moonlight the photocell can detect a diffuse modulated light of an intensity only 1/13,000 of that of a diffuse flashing light just visible to the eye.

Sources of modulated light

The instantaneous intensity I of modulated light may be expressed as a function of time t by a Fourier expansion

$$I = A_0 + A_1 \sin \omega(t - t_1) + A_2 \sin 2\omega(t - t_2) + \text{etc.}$$

The effectiveness of modulated light when used with a tuned receiver such as we have described, depends upon the amplitude A_1 of the fundamental component and therefore this component should be made as large as possible compared to A_0, A_2, etc.

We may define the degree of modulation as the ratio $A_1:A_0$, that is, the ratio of the amplitude of the fundamental component to that of the average light. This ratio for a series of different wave shapes is given in Fig. 14. The maximum possible degree of modulation is 200 percent which is approached if the light consists of separate flashes of very short duration.

An efficient source of modulated light has been developed* in the form of a self-modulated neon lamp which operates on the thyratron principle, and produces the light flashes illustrated in diagram *g* of Fig. 14, giving a degree of modulation as high as 188 percent.

Diffusion of Light in Fog

The aviator encounters several conditions which cause poor visibility. Haze and smoke are usually due to the presence of extremely small particles in the air—particles of diameters less than the wave-length of light. According to Rayleigh's law the amount of light scattered by such particles is inversely proportional to the fourth power of the wave-length of the light. Thus white light transmitted through haze or smoke becomes red while the light scattered is mainly blue. Scattering of this kind accounts for the blue color of the sky and of distant mountains and for the red color of the setting sun.

Besides scattering light, smoke and smoky fog obviously absorb a considerable amount of light.

In districts far from the sources of industrial smoke, however, the aviator encounters clean white fog, either as ground fog or as clouds. Such fog consists of small water drops which range in diameter** from about 2.2×10^{-3} to about 10^{-2} cm, so that they are 40 to 200 times larger than the wave-length of ordinary light. The color effects characteristic of haze and smoke are thus absent in clean fog and in clouds.

The question is often raised whether certain colors of light such as red or blue are not transmitted through fog far better than white light.[6] The large

* This modulated light source will be described in a forthcoming paper by W. F. Westendorp.

** Handbuch d. Exp. Physik **25** (1) p. 51 (1928).

[6] See article on Fog Penetration, by L. H. Anderson, U. S. Air Services, July 1931, p. 20–25. This is to appear in Trans. Amer. Soc. Mech. Eng.; Section on Aeronautical Engineering.

size of the drops indicates that the scattering of light by fog should be the same throughout the visible and ultraviolet spectrum and in the infrared a decrease in scattering should occur only when wave-lengths of about 20 microns are reached. In the visible spectrum this conclusion is confirmed by the fact that the disk of the sun when faintly discernible through clean fog or cloud appears colorless; also by the fact that the illumination on cloudy days is not of distinctly different color from that on clear days.

The scattering of light by fog and clouds is thus due to the interception of the light rays by the spherical droplets of water. Let n be the number of droplets per unit volume, each of diameter d, then w the weight of the drops per unit volume is

$$w = (\pi/6)nd^3. \tag{22}$$

If a beam of light of intensity I passes a distance dx through the cloud the decrease in the intensity of the direct beam is $-dI = I(\pi/4)nd^2dx$ since $(\pi/4)d^2$ is the area intercepted by each drop. The scattering coefficient is thus $(\pi/4)nd^2$. The reciprocal of this or

$$\lambda_0 = 4/(\pi nd^2) = \frac{2}{3}\frac{d}{w} \tag{23}$$

is the "free path" of the light rays through the cloud, i.e., the average distance that the rays can travel before the intensity of the direct beam falls to $1/e$th value. This free path is entirely analogous to that employed in the kinetic theory of gases.

Experimental determinations of w in rain clouds have given 1×10^{-6} to 2×10^{-6} g·cm^{-3}. An estimate of the maximum possible values of w can be made from the water vapor content of air.

The air rising into a rain cloud is saturated when it reaches the level of the bottom of the cloud. Under summer conditions the temperature at this point (1000 m elevation) may be taken to be 20°C. The total water vapor content is then 17.2×10^{-6} g·cm^{-3}. If this saturated air rises another 1000 m it would cool by adiabatic expansion (allowing for the heat of condensation of the water vapor) to 15.8° and the water vapor content would then be 13.3×10^{-6} g·cm^{-3}. The difference between these quantities, i.e., $w = 3.9\times10^{-6}$, must be the volume of the droplets per cm^3 in the cloud at an altitude of 2000 m. A similar calculation for winter conditions with a temperature of -5°C for the base of the cloud gives a temperature of -11.6° at an altitude of 2000 m which gives at this level $w = 1.4\times10^{-6}$. These values of w are in good agreement with the experimentally found values.

The shortest free paths which may occur in clouds or clean fogs thus correspond roughly to $w = 2\times10^{-6}$ and $d = 3\times10^{-3}$ which give by Eq. (23) $\lambda_0 = 1000$ cm or 10 meters. According to Eq. (22) there are only 140 droplets per cm^3 in a fog of this character.

In the less dense clouds and fogs usually encountered by the aviator the free paths λ_0 will range from 30 to 100 meters.

The light which is intercepted by a droplet is scattered in various directions but not uniformly, for the larger part is scattered in a forward direction. This means that the *forward component* of the light originally in a parallel beam does not decrease as rapidly as does the intensity of the unscattered component of the beam. This phenomenon corresponds exactly to that of the *persistence of velocities* treated in the kinetic theory of gases.[7]

The distance at which the sharp outlines of an object can be seen through a fog depends primarily upon the distance that a ray of light can travel without appreciable change in direction. Thus the free path λ_0 that we have calculated from the diameter of the droplets is applicable to such problems.

However if we are to consider the amount of diffuse light transmitted through or reflected from a cloud layer of given thickness we need to consider a different kind of free path λ, i.e., the average distance which the light from a parallel beam can travel before its *forward component* has fallen to $1/e$th value. This free path will probably be two or three times as great as λ_0.

The transmission of light through fog is governed by laws simliar to those of the diffusion of one gas into another and is analogous to the flow of heat or electricity in a conducting solid. The intensity of the light at any point in a fog is approximately the same in all directions and is measured by H the brightness of illumination in candles cm^{-2}.

Corresponding to this average brightness H there is a *random flux density* $F=\pi H$ of light flowing across an imaginary plane inclined in any direction. However, because H is not uniform throughout the fog there will be a *drift flux density* F_d which represents the net flow across an imaginary plane surface. This is essentially a vector quantity and its component in any direction is proportional to the gradient of H in that direction.

The coefficient of self diffusion D in a gas may be calculated from the mean free path λ and the average molecular velocity v, giving the result $D=\lambda v/3$. An analogous calculation for the diffusion of light leads to the following expression for F_x, the component of the drift flux in the direction parallel to the x axis.

$$F_x = -(4\pi/3)\lambda \frac{\partial H}{\partial x} . \tag{24}$$

Thus the quantity $4\pi\lambda/3$ may be termed the *luminous conductivity* of the fog. The brightness H is a scalar quantity which may be regarded as the *luminous potential* and is entirely analogous to electric potential, to temperature in heat conduction problems, or to concentration in diffusion problems.

In deriving the Eq. (24) it was assumed that the free path λ was small compared to the dimensions of the cloud.

With this point of view let us now calculate the distribution of brightness in certain typical simple idealized cases.

[7] See Jeans, Dynamical Theory of Gases.

1. Point source at center of spherical cloud

Consider a source of L lumens at the center O of a cloud of radius r_0. Since we are assuming that the light is scattered, but not absorbed, the total flux across any spherical surface having its center at O is L. Thus the light flux density F_r in a radial direction at any distance r from the center is

$$F_r = L/4\pi r^2. \tag{25}$$

Combining this with Eq. (24) and integrating we find

$$H = \frac{3L}{16\pi^2\lambda}\left(\frac{1}{r} + K\right)$$

where K is the integration constant. Since we have assumed that λ is small compared to r_0, we may put $H=0$ when $r=r_0$ (the outer surface of the cloud) and thus find $K=-1/r_0$, so that

$$H = \frac{3L}{16\pi^2\lambda}\left(\frac{1}{r} - \frac{1}{r_0}\right). \tag{26}$$

Let us examine more closely these boundary conditions. As seen from a distance, the outside surface of the cloud will appear to have a brightness H_0 which may be calculated by Eq. (3) from the flux density F given by Eq. (25) when $r=r_0$. Thus

$$H_0 = L/4\pi^2 r_0^2. \tag{27}$$

An observer located at the outside surface of the cloud would see this brightness H_0 if he faces the interior of the cloud but finds the brightness $H=0$ if he faces away from the center. Thus at this surface H is no longer a scalar quantity as it is in the interior of the cloud but depends on the direction of observation. If we put in Eq. (26) $H=H_0$ as given by Eq. (27) we find that

$$r_0 - r = (4/3)\lambda. \tag{28}$$

In other words, the brightness at a distance $(4/3)\lambda$ inside of the outer surface of the cloud has a value (H_0) which will account for the light flux emanating from the cloud. This result appears reasonable when we remember that the light that escapes from the cloud passes through a layer comparable with λ in thickness without appreciable scattering.

In general, therefore, in calculating the brightness distribution within any cloud mass we may place $H=0$ at the outer surface. The apparent brightness as seen from a point outside is then the brightness, which is calculated for a surface lying at a depth $(4/3)\lambda$ inside of the true surface.

It is interesting to compare the brightness H inside of an illuminated cloud with the brightness which would be obtained at the same distance in the absence of the fog if the light source should shine directly on a white diffusing surface. Comparing H as given by Eq. (26) with $H=L/4\pi^2r^2$ which

would apply in the absence of fog, we see that the effect of the fog is to *increase* the brightness in the ratio

$$3r(r_0 - r) : 4\lambda r_0. \tag{29}$$

As r increases this ratio reaches a maximum value $3r_0/16\lambda$ when $r = 0.5\ r_0$. Thus in a spherical cloud having a radius of 1000 m, and a free path $\lambda = 30$ m, a 5000 candle power light ($L = 60{,}000$ lumens) at the center would give a brightness $H = 3.8 \times 10^{-6}$ candles $\times$ cm^{-2} at a distance of 500 m from the source. This is over six times the brightness (0.63×10^{-6}) that would be obtained if the light from the source at this distance fell directly with normal incidence on a white diffusing surface. The fog, because of its poor luminous conductivity, thus tends to prevent the escape of the light and so increases the brightness within the cloud. The brightness of the outside surface of the cloud, by Eq. (27), would be $H_0 = 1.5 \times 10^{-7}$ c·cm^{-2} which is about 4000 times the threshold sensitiveness of the photoelectric receiver.

Except by means of rockets or flares we are not ordinarily able to produce light sources at the centers of large clouds so the foregoing case may seem to be rather academic. However, if the ground reflects nearly all the light that falls on it, as for example when it is snow covered, then with a thick layer of fog over the ground and a light source on the ground the drift flux distribution is twice that given by Eq. (25) and therefore Eq. (26) gives the brightness distribution if its right side is multiplied by 2. We shall see that imperfect reflection from the ground modifies these conditions.

Directional effect in fog

The drift flux density F_x in a fog as given by Eq. (24) may be regarded as the difference of two fluxes flowing in opposite directions. Any small plane white surface (of dimensions small compared to λ) placed in the fog in a plane perpendicular to the x-axis will be illuminated differently on its two sides, the differences of brightness ΔH_x being

$$\Delta H_x = \frac{F_x}{\pi}. \tag{30}$$

Substituting into this the value of F_r from Eq. (25) and dividing by H as given in Eq. (26) we find for the *directional ratio*

$$\frac{\Delta H_r}{H} = 4\lambda/3r(1 - r/r_0). \tag{31}$$

The minimum value of this quotient, $16\lambda/3r_0$, occurs when $r = 0.5\ r_0$. Thus in the example we have just considered the smallest value of $\Delta H_r/H$ is 0.16. The differential photoelectric receiver, which has been described, can detect differences of only 1.5 percent, and should therefore be able to indicate the direction from which the light comes.

If such a device were used in an airplane the plane could be turned to-

wards the light of greatest intensity until the two intensities balance; straight flight would then bring the plane to the source.

2. Layer of cloud uniformly illuminated from above or below

Let us consider a layer of cloud of uniform thickness h, the upper surface of which is illuminated by sunlight or moonlight. If F_s is the flux density produced by the source as given for example in Table I then if the upper surface of the cloud were a perfect diffuse reflector its brightness would be

$$H_s = (F_s/\pi) \sin \alpha \tag{32}$$

where α is the angular altitude of the source above the horizon. Let H_T be the actual brightness of the *top* of the cloud layer which is less than H_s because of light lost by transmission through the cloud. Let H_B be the brightness of the *bottom* of the cloud layer as seen from the ground, and let R be the average reflection coefficient of the ground. The corresponding flux densities ($F=\pi H$) across horizontal planes are F_T, and F_B. The light flux reaching the ground is thus F_B, while that returning from the ground is RF_B and the loss of light caused by absorption by the ground is $F_B(1-R)$. This loss must be equal to the drift flux F_z through the cloud layer and must also be equal to $F_s \sin \alpha - F_T$, the net light input at the top of the cloud layer. Thus by Eqs. (24) and (32) we have

$$H_B(1 - R) = \frac{4\lambda}{3h}(H_T - H_B) \tag{33}$$

and

$$H_B(1 - R) = H_s - H_T. \tag{34}$$

Separating the two variables H_T and H_B, we obtain the equations

$$\frac{3h}{4\lambda} = \frac{H_T}{H_s - H_T} - \frac{1}{1 - R} \tag{35}$$

and

$$\frac{3h}{4\lambda} = \frac{H_s - H_B}{H_B(1 - R)} - 1. \tag{36}$$

Photometric measurements of H_T and H_B could readily be made from an airplane and by simultaneous determinations of h, the values of λ for various types of cloud layers could be found. Measurements from the ground alone, however, give some useful data. Published tables of photographic exposure show that on days described as "very dull" the exposure only needs to be increased about ten-fold. If under these conditions the upper surface of the cloud layer reflects 90 percent of the incident light there can be no appreciable loss by absorption within the cloud layer. The effect of a cloud layer sufficient to make a very dull day, which we may assume is one or two thousand meters thick, is thus to change the light flux on the ground in the ratio $H_B:H_s=1:10$. From Eq. (36) this gives

$$\frac{3h}{4\lambda} = \frac{8 + R}{1 - R}.$$

Photometer measurements which we have made from a tower at an airport have shown that the reflectivity R of the ground is ordinarily very low; for example the airport surface varied from 9 to 21 percent reflectivity. Tree covered land (without leaves) gave only 5 percent. The foregoing expression gives values of h/λ which are not very sensitive to R; thus for $R=0.1$ and $R=0.2$, we find h/λ is 12 and 13.7 respectively. This gives $\lambda=80$ m with a 1000 m layer or $\lambda=160$ m with a 2000 m layer.

If we have a layer of cloud uniformly illuminated from below as for example by a series of regularly spaced lights, the problem differs from that just considered in that the light reflected downwards, F_B, from the bottom of the cloud layer is partly reflected back by the ground and contributes to the illumination of the cloud. Equation (34) is still applicable but in place of Eq. (33) we have

$$H_T = \frac{4\lambda}{3h}(H_B - H_T) \tag{33a}$$

and instead of Eqs. (35) and (36) we obtain

$$1 + \frac{3h}{4\lambda} = \frac{H_s - H_T}{H_T(1 - R)} \tag{35a}$$

and

$$\frac{3h}{4\lambda} = \frac{H_B(2 - R) - H_s}{H_s - H_B(1 - R)} \tag{36a}$$

where H_s is the brightness which the bottom of the cloud would have if it were perfectly reflecting and the ground were completely nonreflecting. If F_s is the average vertical flux density from the light sources on the ground then $H_s=F_s/\pi$. It should be noted (by Eq. (36a)) that if the cloud layer is thick or λ is very small so that $3h/4\lambda$ becomes very large, H_B approaches the value

$$H_B = H_s/(1 - R)$$

and thus may become much greater than H_s if the ground has high reflectivity.

3. Point source in fog close to ground with poor reflectivity

We have already considered in Case I an example where the ground is a good reflector. Let us now consider a case where the ground is completely black ($R=0$), and the light source is located at a point B at a height b above the ground.

Since the ground absorbs all the light that falls on it, $H=0$ at this surface just as at the outside surface of the fog. We can simplify our problem by as-

suming that the fog is infinitely thick. The problem is then similar to that of the potential distribution around a point charge at a distance b from a plane conducting surface, and may be readily solved by the method of images. In the present case the distribution is the same as if the black ground were replaced by a completely transparent plane and in addition to the light source at B we postulate a light sink (negative source) at B' which is the image of B in the plane. The brightness H at any point at a distance r from the light source ($r \gg \lambda$) and at an angular height α above the horizon (as seen from the source) is

$$H = \frac{3L_u b}{4\pi^2 \lambda r^2} \sin \alpha. \tag{37}$$

In this equation L_u represents the light flux in lumens emitted by the source in an *upward* direction, i.e., in the upper hemisphere. This distinction is necessary since rays emitted downward would be absorbed directly by the ground. In practice the efficiency of the light source can be nearly doubled by placing a hemispherical reflector below the lamp to convert the downward rays into the useful upward light. In this case L_u is approximately equal to L the total luminous output of the lamp.

If the light source were actually at the level of the ground ($b=0$) Eq. (37) would give $H=0$. When we consider that the light radiating out from the source reaches on the average a hemispherical surface of radius λ before being scattered, we see that the effective height of a point source on the ground is roughly $2\lambda/3$. If the light were focussed vertically upwards its effective height† would more probably be λ. Thus b in Eq. (37) may be taken to be $b_0+(2/3)\lambda$ (or $b_0+\lambda$) if b_0 is the actual height of the light source.

With $b_0=0$ so that $b=(2/3)\lambda$ Eq. (37) thus becomes

$$H = L_u \sin \alpha / 2\pi^2 r^2. \tag{38}$$

If there were no fog, and the light source should shine directly with normal incidence on a white surface at distance r the brightness would be

$$H = L_u / 2\pi^2 r^2 \tag{38a}$$

which is thus the same as the brightness within the fog at all points directly over the source. For other values of α the brightness would be less.

Fig. 15 gives data on the range at which a modulated light of intensity $L_u=60{,}000$ lumens (5000 spherical candles with hemispherical reflector) would produce in an infinite fog a brightness $H=3\times10^{-9}$ c·cm^{-2}, this being about 75 times the threshold value of the photoelectric receiver.

Curve I is for the case that the beacon is at the ground level (effective height $2\lambda/3$). The range is then independent of the density of the fog. Curves II and III illustrate the effect of placing the beacon on a tower, 50 m high.

† A still greater effective height might be obtained in quiet foggy weather by building a fire on the ground so as to produce a rising vertical column of clear air up through which a search light beam could be projected to a height much greater than λ

The range is greatly increased and is now dependent on the density of the fog, being much greater for a dense fog than for one less dense. With no fog at all the range (for $H = 3\times10^{-9}$) is represented by Curve IV which by Eq. (38a) is a semicircle of radius equal to the maximum height of Curve I. Comparison of Curves III and IV shows how greatly fog can *extend the range* of diffuse light signals.

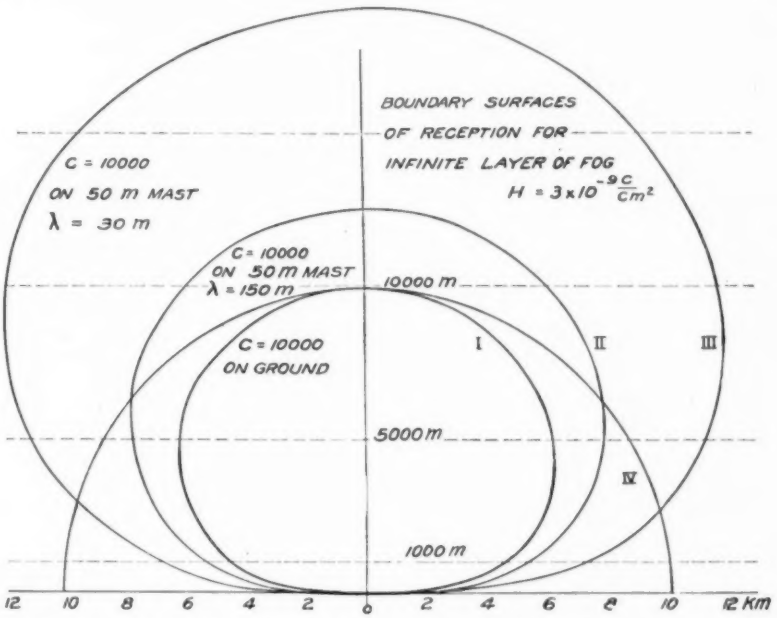

Fig. 15. Surfaces of equal brightness ($H = 3\times10^{-9}$ c·cm^{-2}) in an infinite fog overlying dark colored ground ($R=0$) near which is a light source projecting 60,000 lumens into the upper hemisphere; C is the candle power in the upward direction, i.e., of the lamp with its hemispherical reflector.

If we take x as the horizontal distance from the light source ($x = r\cos\alpha$) then by partial differentiation of Eq. (37) and combination with Eq. (24) we find that the horizontal component of the drift flux density, at any point, is

$$F_x = \frac{3L_u b}{\pi r^3}\cdot\sin\alpha\cos\alpha \tag{39}$$

and thus by Eq. (30) the directional ratio is

$$\frac{\Delta H_x}{H} = (4\lambda/r)\cos\alpha \tag{40}$$

which is independent of b, the height of the light source.

If the receiver can detect a difference of 1.5 percent between opposite directions, it should be capable of giving directional effects at distances of about 200λ or 10 km (if H is as great as 3×10^{-9}).

4. Point sources within uniform layer of fog or cloud

Case a—White ground ($R=1$)—Consider a point source of light on perfectly reflecting ground which is covered by a layer of fog of thickness h. At the upper surface of the fog $H=0$. By the method of images the problem is then equivalent to the determination of the potential distribution produced by a series of alternate positive and negative charges arranged along a vertical line with a distance $2h$ between adjacent charges. Madelung has solved[8] this and many similar problems in his studies of the electric forces in crystals built up of ions. The complete solutions are expressible only in terms of Hankel functions for which, however, tables are available. We are interested mainly in the distribution of brightness at distances from the source which are fairly large compared to h the thickness of the cloud layer. Under these conditions the equations take much simpler forms. The following equations are modifications of Madelung's results adapted to our calculation of distribution of brightness.

At any point within the fog layer at a height y above the ground and a horizontal distance x from the beacon the brightness H is

$$H = \frac{3L \exp(-\pi x/2h) \cdot \cos(\pi y/2h)}{4\pi^2 \lambda h^{1/2} x^{1/2}}. \tag{41}$$

Here L is the total lumens output from the light source. Because of the perfect reflection from the ground all of this light escapes from the upper surface of the cloud layer. This equation is accurate for all values of x which are large compared to h. However, for values of x even as small as $h/3$ the errors are not greater than about 10 percent.

A calculation of the directional ratio similar to that which led to Eq. (40) gives in the present case for $x \gg h$

$$\frac{\Delta H_x}{H} = 2\pi\lambda/3h. \tag{42}$$

The brightness H_0 of the upper surface of the fog, at a point directly over the beacon, $x=0$, $y=h$, is (exactly)

$$H_0 = 0.09280L/h^2. \tag{43}$$

At points on the upper surface for which $x>h$, the brightness H_0 is

$$H_0 = \frac{L \exp(-\pi x/2h)}{2\pi h^{3/2} x^{1/2}}. \tag{44}$$

Examination of these results shows that the brightness in the fog and on its upper surface decreases exponentially as the horizontal distance x increases, falling to $1/e$th value in each increment of $0.63h$ in the distance.

As an example of the range at which modulated light signals can be detected we may assume the following conditions. An airplane flies at a height

[8] E. Madelung, Phys. Zeits. **19**, 524–532 (1918).

y of 250 m in a fog layer $h=500$ m thick, with a free path $\lambda=50$ m. The ground is snow-covered and on the ground is a modulated light beacon of 5000 candle power, $L=60{,}000$. By Eq. (41) we find that the brightness in $c\cdot cm^{-2}$ at various horizontal distances x from the beacon is 1.2×10^{-6} at 1 km, 3.7×10^{-8} at 2 km, 1.3×10^{-9} at 3 km, and 5×10^{-11} at 4 km. Thus since 4×10^{-11} $c\cdot cm^{-2}$ can be detected, the signal could be picked up at a distance of 3 to 4 km from the source. The directional ratio of about 20 percent should give good directional effects at distances less than 3 km. The brightness H_0 at the top of the cloud layer is $3\lambda/h$ or 0.3 of that at the height of 250 m so that the ranges of detection just found are approximately independent of the height of the plane within the layer.

Case b—Black ground. $R=0$. By the method of images this problem is equivalent to that of finding the potential of a series of dipoles arranged in a vertical line with uniform spacing of $2h$. Each dipole consists of an upper positive source L and a lower negative source $-L$ with a distance $2b$ between them.

If the height of the source above ground is small compared to h the thickness of the fog layer, then the brightness H at any point (x, y) is

$$H = \frac{3L_u b}{2^{3/2}\pi\lambda h^{3/2}x^{1/2}} \sin\left(\frac{\pi y}{h}\right) \exp\left(-\pi x/h\right) \tag{45}$$

L_u is again the total light thrown *upwards* by the beacon.

As in Eq. (37), b is the effective height $b_0+(2/3)\lambda$, of the beacon.

Thus if the beacon is practically at the ground level so that $b=(2/3)\lambda$ we have

$$H = \frac{L_u \sin\left(\pi y/h\right)\cdot\exp\left(-\pi x/h\right)}{2^{1/2}\pi h^{3/2}x^{1/2}}. \tag{46}$$

A calculation of the directional ratio gives for $x\gg h$

$$\frac{\Delta H_x}{H} = 4\pi\lambda/3h \tag{47}$$

which is twice as great as that given by Eq. (42) for $R=0$.

The brightness H_0 at the top surface of the fog directly over the beacon is

$$H_0 = 0.4262bL_u/h^3. \tag{48}$$

At horizontal distances x large compared to h the brightness of the top surface is

$$H_0 = \frac{2^{3/2}L_u\lambda \exp\left(-\pi x/h\right)}{3h^{5/2}x^{1/2}}. \tag{49}$$

A rough integration of the total light flux L_0 in lumens from the upper surface of the fog gives

$$L_0 = 2L_u b/h. \tag{50}$$

It should be noted that the logarithmic decrement in the brightness as x increases is twice as great as in the case that the ground is white.

As an example let us take as before a modulated light beacon of 5000 candles ($L_u = 60{,}000$). For various values of h and x we have calculated H at the height $y = 0.5h$ by Eq. (46) for the case that the beacon is close to the ground. Table VIII gives the distances at which the brightness H has the values specified in Column I. With the photoelectric receiver capable of detecting 4×10^{-11} c·cm^{-2} ranges of 1 to 3 km are possible. At heights other than $0.5h$ the results are not greatly different.

TABLE VIII. *Distances in meters at which the light from a 5000 candle power beacon (with reflector) will produce given brightness of illumination in the interior of a fog layer of thickness h, covering dark colored ground.*

H c·cm^{-2}	$h=250$ m	$h=500$ m	$h=1000$
10^{-8}	580	950	1500
10^{-9}	760	1300	2180
10^{-10}	930	1640	2870
4×10^{-11}	1000	1780	3140

A ten-fold increase in the output of the beacon or a ten-fold increase in the effective height b would produce only a 20 or 30 percent increase in the range at which the light from the beacon could be detected by the photoelectric receiver.

Comparing these data with those calculated for the perfectly reflecting ground we see that the effect of making the ground black is to reduce the range to about 42 percent of its value for perfectly reflecting ground.

5. Reflection from and transmission through a cloud layer

The case that we have just considered of a thin layer of fog in contact with the ground is probably the most unfavorable that can be found for light transmission inside of the fog layer. If, however, the aviator flies above the fog layer he can make use of the spot of light which is transmitted up through the fog according to Eq. (50). If the upper surface of the fog is horizontal the light flux emitted from this spot in a direction inclined at an angle α above the horizon will be proportional to $\sin\alpha$. The light flux density F at any point which is at the distance r (along inclined line) from the center of the spot and at the height y above the top of the fog layer is thus

$$F = L_0 y/\pi r^3 = 2L_u b y/\pi h r^3. \tag{51}$$

The range of the signals that may be transmitted in this way is far greater than those within the fog layer itself. For example, putting $L_u = 60000$, $b = (2/3)\lambda$, $\lambda = 50$m, $h = 250$m, we have $F = 5100\ y/r^3$. If we take $y = 2.5\times10^4$, (250m) and $F = 1.2\times10^{-10}$ corresponding to the threshold ($H = 4\times10^{-11}$) of the photoelectric receiver we see that we should be able to detect the modulated light transmitted up through the fog at a distance of 10.2 km as compared to the distance of only 1 km within the fog layer as given in Table VIII.

If there are other layers of cloud above the fog on the ground the light reflected down from them will greatly increase the range and will also make it possible to detect the modulated light within the fog layer at distances far greater than those given in Table VIII.

If the bottom of the fog layer is at a height y_0 above the ground then the light L_u from the beacon forms a spot of light on the bottom of the cloud which reflects the greater part of the total light. Thus if the plane flies below the cloud layer the range may be calculated by the first two members of Eq. (51), where L_0 is now to be replaced by L_u and y is the height of the *ceiling above the plane.*

When the cloud layer is definitely off the ground the transmission of light through it is given by Eqs. (35a) and (36a). The fraction of the light output of the beacon which escapes from the top of the cloud layer is equal to H_T/H_s. By Eq. (35a) this is

$$H_T/H_s = 1/[1 + (1 - R)(1 + 3h/4\lambda)]. \tag{52}$$

When $R=0$ and h/λ is large this reduces to the same value as the ratio L_0/L_u given by Eq. (50) for the case of the fog lying on black ground. Equation (51) is thus still applicable for the detection of light transmitted through a cloud layer which is at a considerable height above ground.

In this case the brightness within the cloud layer will extend for relatively large horizontal distances from the source. The brightness H at any point may be calculated by linear interpolation between H_B and H_T which are given by Eqs. (35a) and (36a). The brightness H_s which is to be used for this purpose is equal to

$$H_s = L_u y_0/2\pi^2 r^3 \tag{53}$$

where r is the inclined distance from the light source to the given point at the bottom of the cloud. In most applications where $r \gg y$ we may put $r=x$, the horizontal distance from the source.

Under these conditions we find by partial differentiation of Eq. (53) and combining with Eqs. (24) and (30) that the directional ratio is

$$\frac{\Delta H_x}{H} = \frac{4\lambda}{x} \tag{54}$$

which is $3h/\pi x$ times as great as that given by Eq. (47) for the ratio inside of a fog layer on the ground.

As an example let us consider a cloud layer of a thickness of $h=500$m whose bottom surface is $y_0=500$m above the ground which has the reflectivity $R=0.2$. Let $\lambda=50$m and $L_u=60{,}000$ lumens. By Eq. (53) we find $H_s=1.5\times10^8/r^3$c·cm^{-2}, and then by Eqs. (35a) and (36a) we get $H_T=1.3\times10^7/r^3$ and $H_B=1.09H_s=1.65\times10^8/r^3$. The directional ratio is $\Delta H_x/H=2\times10^4/x$. Placing $H_T=4\times10^{-11}$ we find that a plane flying near the top of the cloud layer should be able to detect the modulated light at a distance $x=r=6.9$ km. Near the bottom of the cloud layer the range would be 16.1 km. At

distances of 1.65 and 3.8 km respectively (top and bottom) the brightness would be 3×10^{-9} (75 times threshold) and the directional ratios would be 0.121 and 0.053 respectively, whereas 0.015 can be observed with this brightness.

The effect of true absorption of light within a fog

So far we have considered the scattering of light within a clean white fog in which absorption is negligible. It may be of interest to deal briefly with the modifications that occur if there is also an absorption of light such as exists for example in a smoky fog.

In this case, in each unit of volume, an amount of light will be absorbed (destroyed as distinguished from scattered) which will be proportional to the light intensity H at that point. The absorption may be measured in terms of a volume-absorption coefficient, A, which is the light absorbed per unit volume per unit light intensity. Thus AH lumens of light are absorbed per unit of volume.

The absorbing power of the fog may also be described in terms of a mean free path λ_A. Perhaps the best way of defining this quantity is by means of the relation

$$-\frac{dF}{F}=\frac{dx}{\lambda_A} \tag{55}$$

where $-dF$ is the decrease in the intensity of a beam of light of flux density F, resulting from absorption, while travelling a distance dx.

It can be shown that

$$\lambda_A = 4\pi/A\,. \tag{56}$$

Let us now consider three special cases in which H depends only on a single coordinate, thus H depends upon: (1) the distance r from a fixed plane, $\beta=0$. (2) the distance r from a fixed line, $\beta=1$. (3) the distance r from a fixed point, $\beta=2$. Here β is a number that has the value 1, 2 or 3 for the three cases.

Then by Poisson's equation we have for all three cases

$$\frac{d^2H}{dr^2}+\frac{\beta}{r}\frac{dH}{dr}=\frac{3H}{\lambda_A\lambda_s} \tag{57}$$

where λ_s is the free path for scattering, i.e., the quantity which we have previously designated by λ.

If we impose the boundary condition that $H=0$ at $r=\infty$, the integration of Eq. (57) gives for:

1. *Plane case*, $\beta=0$

$$H = H_0 \exp\left[-(3/\lambda_A\lambda_s)^{1/2}r\right] \tag{58}$$

where H_0 is the brightness of illumination at the fixed plane, $r=0$. Differentiation and substitution into Eq. (24) gives the flux density F_0 at $r=0$,

$$F_0 = 4\pi(\lambda_s/3\lambda_A)^{1/2}H_0. \tag{59}$$

2. *Cylindrical case*, $\beta = 1$. The exact expression for this case involves Bessel functions. However, in practical cases where $(3/\lambda_s\lambda_A)^{1/2}r$ is larger than unity, a sufficiently close approximation is given by

$$H = 0.03618\lambda_A^{1/4}\lambda_s^{-3/4}r^{-1/2}L_1 \exp\left[-(3/\lambda_A\lambda_s)^{1/2}r\right] \tag{60}$$

where L_1 is the light output from the line source in lumens per unit length.

3. *Spherical case*, $\beta = 2$

$$H = (3L/16\pi^2\lambda_s r) \exp\left[-(3/\lambda_A\lambda_s)^{1/2}r\right] \tag{61}$$

where L is the total output from the point source in lumens.

In the absence of absorption Eq. (26) gives

$$H = 3L/16\pi^2\lambda_s r. \tag{62}$$

Comparing Eqs. (58), (60) and (61) with (62) the combined effect of scattering and absorption is seen to give *an effective free path* equal to 0.577 $(\lambda_A\lambda_s)^{1/2}$. Because of the presence of the exponential factor in these equations, it is obvious that useful signals can never be transmitted to distances greater than about $10\times(\lambda_A\lambda_s)^{1/2}$.

We have now considered the various factors which seem to be of greatest importance in determining the transmission of diffuse light signals from ground to an airplane. It seems possible to draw the definite conclusion that with sources of reasonable intensity and with photoelectric detectors which can now be built, signals may be transmitted considerable distances through dense white fog. The presence of smoke may greatly decrease the range.

CAPILLARY CONDUCTION OF LIQUIDS THROUGH POROUS MEDIUMS

By L. A. Richards
Cornell University

(Received April 16, 1931)

Abstract

The flow of liquids in unsaturated porous mediums follows the ordinary laws of hydrodynamics, the motion being produced by gravity and the pressure gradient force acting in the liquid. By making use of Darcey's law, that flow is proportional to the forces producing flow, the equation $K\nabla^2\psi+\nabla K\cdot\nabla\psi+g\partial K/\partial z=-\rho_s A\partial\psi/\partial t$ may be derived for the capillary conduction of liquids in porous mediums. It is possible experimentally to determine the capillary potential $\psi=\int dp/\rho$, the capillary conductivity K, which is defined by the flow equation $q=K(g-\nabla\psi)$, and the capillary capacity A, which is the rate of change of the liquid content of the medium with respect to ψ. These variables are analogous, respectively, to the temperature, thermal conductivity, and thermal capacity in the case of heat flow. Data are presented and application of the equations is made for the capillary conduction of water through soil and clay but the mathematical formulations and the experimental methods developed may be used to express capillary flow for other liquids and mediums. The possible existance of a hysteresis effect between the capillary potential and moisture content of a porous medium is considered.

Introduction

THE term capillarity initially referred to the action of liquids in fine tubes but has since come to be used in connection with a wide variety of phenomena, including under certain conditions the flow of liquids through porous mediums.

Buckingham[1] was among the first to attempt a detailed analysis of capillary flow. He assumed capillary attraction to constitute a conservative force field and defined a capillary potential, the gradient of which was equal to the capillary force. Some years later Gardner and his associates[2] pointed out that the Buckingham potential was closely related to the pressure in the water films and showed that porous clay apparatus could be used for its measurement. A discussion of the relation between the so-called capillary force and the pressure in the film liquid has been given by the writer[3] and experimental data were presented to show that if a porous medium is wetted by a liquid and if the medium is unsaturated, the pressure in the liquid films is less than atmospheric pressure and depends on the amount of liquid present in the medium.

Whenever there is a difference in pressure between two points in a liquid film there is a tendency for water to move in the direction of the decrease in

[1] E. Buckingham, U. S. D. A. Bur. of Soils Bul. **38**, 61 (1907).
[2] Gardner, Israelsen, Edlefsen and Clyde, Phys. Rev. **20**, 196 (1922).
[3] L. A. Richards, Jour. of Ag. Res. **20**, 719 (1928).

pressure. A study of this phase of the problem indicates that the capillary conduction of liquids through porous mediums is similar to the flow of liquids through pipes or fine tubes to the extent that in both cases the flow may be expressed in terms of gravity and the pressure gradient in the liquid. There is, however, a significant difference between the two classes of flow phenomena and we are quite justified in continuing to use the term capillarity when dealing with liquid systems where the pressure distribution is determined by the surface tension and curvature of a gas-liquid interface.

Among the expressions which have been derived for capillary flow, those given by Gardner[4,5] are the most noteworthy but as yet a general theory in terms of known or measurable functions has not been published. The problem is complicated by a large number of variables and the complex configuration of the region occupied by the capillary liquid has been a stumbling block for theoretical analysis. Also, the lack of suitable apparatus has prevented accurate experimental measurements. In the present work some of the experimental difficulties have been overcome and the problem has been approached by methods which are closely analogous, both in theory and experiment, to those used in the study of thermal and electrical conduction in metals.

Forces Affecting Capillary Action

The forces acting in the boundary surfaces of liquids are directly responsible for all capillary phenomena and have their origin in the cohesive and adhesive attractions which are exerted between molecules. The most common cases of capillary action occur when one surface of a liquid is in contact with air. Because of the free energy possessed by the molecules of the liquid lying in such a surface there is a tendency for the surface to assume that configuration which makes the area a minimum. Mathematically a uniform surface density of free energy is equivalent to a uniform tension in the surface and for some purposes it is more convenient to express the physical properties of an air-liquid surface in terms of surface tension.

For instance, the pressure difference caused by a curved liquid surface is usually expressed by the relation,

$$p_W - p_A = T_1(1/R_1 + 1/R_2), \tag{1}$$

p_W and p_A being the pressures on the two sides of the surface, T_1 the surface tension, and $(1/R_1+1/R_2)$ the total curvature of the surface.

Because of the inequality between cohesive and adhesive forces at a liquid-solid interface, these surfaces also have a free energy which is important in determining the capillary behavior of liquids. If, for a given liquid and solid the surface densities of free energy for the air-liquid, air-solid and liquid-solid interfaces are respectively T_1, T_2, and T_{12}, it may be shown[6] that,

[4] W. Gardner, Soil Sci. **10,** 103 (1920).
[5] W. Gardner and J. A. Widtsoe, Soil Sci. **11,** 215 (1921).
[6] N. K. Adam, Sci. Progress **21,** 431 (1927).

$$T_1 + T_2 - T_{12} = W = T_1(1 + \cos\theta), \tag{2}$$

W being the work necessary to separate unit area of liquid from solid and θ the angle of contact between liquid and solid. If $W < 2T_1$, θ may vary anywhere between 0° and 180°. If, however, $W \geqq 2T_1$ then θ is zero and the liquid will spread over the surface of the solid and is said to wet the solid.

Because of their effect on the angle of contact, adhesive forces are directly involved in an initial wetting process such as the spreading of a liquid in a dry porous medium, but after the medium is wetted adhesive forces are no longer effective in producing a motion of the liquid and influence capillary action only to the extent that they hold a thin film firmly in contact with the solid surface. The liquid lying outside the adsorbed films is free to move under the action of unbalanced forces. Capillary flow may be expressed in terms of the well-known laws of hydrodynamics and is distinguished from other cases of liquid flow only because of the relation of surface tension and curvature to the pressure and to the effective cross-sectional area of the liquid-transmitting region.

The equation of motion for a viscous fluid is given in various treatises on hydrodynamics[7,8] as,

$$\rho dv/dt = \rho F - \nabla p + \mu(\nabla\nabla\cdot v/3 + \nabla\cdot\nabla v), \tag{3}$$

where ρ is the mass of unit volume of the liquid, dv/dt is the acceleration and the right hand member is the expression for the forces acting per unit volume.

Dividing Eq. (3) by ρ we obtain the relation between the acceleration and forces for unit mass,

$$dv/dt = F - \nabla p/\rho + (\mu/\rho)(\nabla\nabla\cdot v/3 + \nabla\cdot\nabla v). \tag{4}$$

F represents the external or body forces and for most capillary problems gravity is the only external force which need be considered. The term $-\nabla p/\rho$ is the expression for the force due to the pressure gradient and the third term, being a function of the viscosity μ, and the velocity v, is the expression for the viscous retarding forces.

If the gravity force F is represented as the negative gradient of the potential $\phi = gz$, where z is the height from a reference level, then $F = -\nabla\phi = -g$. The variation of the gravity force with z is neglected, g being assumed constant. Also, if ρ is constant or depends on p alone, the force represented by the term $-\nabla p/\rho$ may be expressed as the gradient of the potential $\psi = \int dp/\rho$, the integral being taken from an arbitrarily chosen reference pressure to the pressure at the point in question. Eq. (4) may then be written,

$$dv/dt = -\nabla(\phi + \psi) + (\mu/\rho)(\nabla\nabla\cdot v/3 + \nabla\cdot\nabla v), \tag{5}$$

or,

$$dv/dt = -\nabla\Phi + (\mu/\rho)(\nabla\nabla\cdot v/3 + \nabla\cdot\nabla v), \tag{6}$$

[7] H. Lamb, Hydrodynamics, 5th edition, (1924).

[8] L. Page, Introduction to Theoretical Physics, (1928).

where $\Phi = (\phi + \psi)$ and $-\nabla\Phi$ is the total water-moving field or the total field tending to produce a motion of the water.

Expressions for the flow of viscous liquids through various channels have been derived from the general equations of motion. The equation of Poiseuille derived for flow through small tubes is in excellent agreement with experiment. Applications of the theory have been made by Slichter[9] for the flow of water through saturated porous mediums but the results are less definite in this case, because, even for the simplified idealized system made up of uniform spheres with regular packing, the complicated configuration of the water transmitting region makes necessary the introduction of various simplifying assumptions.

Liquids in Unsaturated Porous Mediums

The pore space region in sand or clay, or even a cotton wick forms a complicated system and the configuration arising when only part of this space is filled with a liquid is even more difficult to picture. If the condition for wetting, as expressed in connection with Eq. (2) is fulfilled, the liquid spreads over the solid surface and forms a continuous and connected configuration. Because of the action of surface tension, liquid tends to collect in small wedge and disk-shaped bodies in sharp corners of the pores or where the particles of the medium are close together, the size of these bodies and the thickness of the films connecting them depending on the amount of liquid present in the medium. It is through this connected configuration, bounded on one side by the adsorbed films in contact with the solid, and on the other by the curved air-liquid interface, that capillary flow takes place. The liquid occupying this region will be spoken of as capillary liquid.

Since water is the only liquid for which capillary flow data have been obtained, the discussion which follows is concerned primarily with water but the mathematical formulations given and the experimental methods presented may be used in expressing the capillary flow of other liquids in porous mediums whenever the condition of zero contact angle is fulfilled.

Pressure in the capillary water of a porous medium is determined by the tension and curvature of the air-water interface as given by Eq. (1), where p_W is the pressure in the water and p_A is the pressure in the air which, in general, will be atmospheric pressure. It is convenient when dealing with capillary phenomena to choose atmospheric pressure as the zero reference. Adopting this convention, p_A may be dropped from Eq. (1) and pressures less than atmospheric pressure will be considered negative.

If the liquid in a porous medium is in contact with free water and is at equilibrium under gravity, then, neglecting the variation in atmospheric pressure and the density of water with height z above the flat water surface, we may write,

$$p_W = T_1(1/R_1 + 1/R_2) = -g\rho z. \tag{7}$$

[9] C. S. Slichter, U. S. G. S. 19th An. Rept. **2**, 301 (1898).

In order for the capillary water to attain the correct pressure for equilibrium at a given height, the moisture content of the medium changes until the curvature has the right value.

If the density of water is assumed to be constant and equal to unity, the potential $\psi = \int dp/\rho$ becomes numerically equal to the pressure. ψ is a potential due to pressure forces and when the pressure is determined by the surface tension and curvature of the air-liquid interface ψ will be called the capillary potential. For a given value of the pressure or capillary potential the amount of moisture present in a medium will depend on the number and kind of pore spaces. The curves in Fig. 1 are reproduced from an earlier paper by the

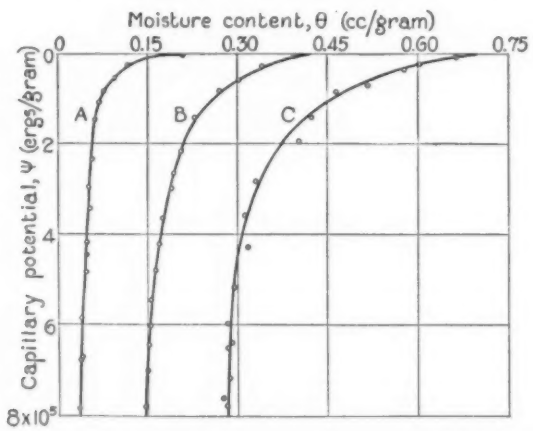

Fig. 1. Curves showing the relation between capillary potential and moisture content for, *A*—Bennet sandy soil, *B*—Greenville loam soil and *C*—Preston clay.

writer[3] and show the relation between moisture content and capillary potential for three mediums. To obtain these data a hollow capillary cell having a flat porous surface was filled with water at a controlled pressure. A layer of the medium to be studied was then spread on the porous surface and the moisture in the medium was allowed to come to pressure equilibrium with the water in the cell. Samples of the medium were removed at each pressure to determine the moisture content. The moisture content is given as the number of cubic centimeters of water per gram of dry medium and will be designated by θ. The capillary potential is expressed in ergs per gram and is numerically equal to the pressure expressed in dynes per square centimeter.

When the conditions for equilibrium under gravity, as expressed by Eq. (7), are fulfilled, the velocity and acceleration of the capillary liquid are everywhere zero and Eq. (5) becomes,

$$\nabla(\phi + \psi) = 0 \tag{8}$$

which means that the force arising from the pressure gradient just balances gravity. If this condition does not obtain there will be a resultant water-moving force and in general there will be capillary flow.

Because of the complex configuration of the capillary liquid it would be

difficult to derive expressions for capillary flow from the general hydrodynamical equations but it is possible to deduce generalizations from experimental data which enable us to set up mathematical relations between the flow and factors causing the flow. Fourier's law and Ohm's law are just such generalizations. An experimental law, quite analogous to these two, and attributed to Darcey, may be used in connection with the present problem.

Darcey, working with mediums under saturated conditions, found that the flow of water through a column of soil is directly proportional to the pressure difference and inversely proportional to the length of the column. For low pressure gradients it has been found by numerous investigators[10,11] that this law is in exact agreement with experiment and it is entirely analogous to the well-known law of Poiseuille for the flow of liquids through capillary tubes. However, both of these laws fail to hold for high pressure gradients. The limits within which they are true and the modifications which a second approximation requires can be determined only by exhaustive experiments on a wide range of materials. In view of the experimental data now available it is assumed that Darcey's law holds for the low velocities and pressure gradients dealt with in this paper.

Mathematically Darcey's law may be expressed by the equation

$$q = -K\nabla\Phi, \tag{9}$$

where q is the volume of water crossing unit area perpendicular to the flow, in unit time and K is a proportionality factor which for a medium whose pore spaces are filled with water will depend on the number and kind of pore spaces and the viscosity. This factor has been variously known as the transmission constant,[9] the coefficient of permeability,[10] and the conductivity.[1] In c.g.s. units it is the number of cubic centimeters of water crossing one square centimeter perpendicular to the flow in one second when there is a water-moving force of one dyne per gram. From analogy with the thermal and electrical cases it will here be called conductivity.

If there is a steady flow of liquid through a porous medium which is only partially saturated, then the larger pore spaces contain air and the effective cross-sectional area of the water conducting region is reduced. If these air spaces could in some way be filled with solid, the condition of the flow would be unchanged and the proportionality between the flow and the water-moving force would still hold because Darcey's law is independent of the size of particles or the state of packing. Hence the essential difference between flow through a porous medium which is saturated and flow through a medium which is unsaturated is that under this latter condition the pressure is determined by capillary forces and the conductivity depends on the moisture content of the medium. The motion of liquids through unsaturated porous mediums will be referred to as capillary flow and for this case K in Eq. (9) will be called capillary conductivity.

[10] Norah D. Stearns, U.S.G.S. Water Supply Paper **596 F,** 121 (1927).
[11] F. H. King, U.S.G.S. 19th An. Rept. **2,** 67 (1898).

The equation of continuity for capillary flow may be written as

$$\nabla \cdot q = -\rho_s \partial\theta/\partial t \tag{10}$$

where $\nabla \cdot q$ is the divergence of the flow, ρ_s is the weight of the dry medium in unit volume and $\partial\theta/\partial t$ is the rate at which the moisture content is changing with the time. Now if θ is a single valued continuous function of ψ, $\partial\theta/\partial t = (d\theta/d\psi)(\partial\psi/\partial t) = A\partial\psi/\partial t$ and Eq. (10) becomes,

$$\nabla \cdot q = -\rho_s A \partial\psi/\partial t. \tag{11}$$

A is the rate of change of the moisture content with respect to the capillary potential and will be called the capillary capacity of the medium. The functional relation of A to ψ or θ may be determined from curves such as those shown in Fig. 1.

Putting Eq. (9) in cartesian coordinates we have,

$$q = -K\nabla(\phi + \psi) = -K\frac{i\partial(\phi+\psi)}{\partial x} + \frac{j\partial(\phi+\psi)}{\partial y} + \frac{k\partial(\phi+\psi)}{\partial z} \tag{12}$$

where i, j, and k are unit vectors along the x, y, and z axes respectively. If the z-axis is chosen positive upward along the vertical then $\partial\phi/\partial x = \partial\phi/\partial y = 0$, $\partial\phi/\partial z = g$, and

$$q = -K[i\partial\psi/\partial x + j\partial\psi/\partial y + k(g + \partial\psi/\partial z)]. \tag{13}$$

Substituting this value of q in Eq. (11) and further transposing to cartesian notation we have,

$$-\left[\frac{\partial K}{\partial x}\frac{\partial\psi}{\partial x} + \frac{K\partial^2\psi}{\partial x^2} + \frac{\partial K}{\partial y}\frac{\partial\psi}{\partial y} + \frac{K\partial^2\psi}{\partial y^2} + \frac{\partial K}{\partial z}\left(g + \frac{\partial\psi}{\partial z}\right) + K\left(\frac{\partial g}{\partial z} + \frac{\partial^2\psi}{\partial z^2}\right)\right] = -\rho_s A\frac{\partial\psi}{\partial t}. \tag{14}$$

Dropping the term $\partial g/\partial z$ and rearranging gives a differential equation for the general case of capillary flow,

$$K\left(\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2}\right) + \frac{\partial K}{\partial x}\frac{\partial\psi}{\partial x} + \frac{\partial K}{\partial y}\frac{\partial\psi}{\partial y} + \frac{\partial K}{\partial z}\frac{\partial\psi}{\partial z} + g\frac{\partial K}{\partial z} = -\rho_s A\frac{\partial\psi}{\partial t}. \tag{15}$$

If from experimental data, K and A can be expressed as functions of the capillary potential, then ψ is the only dependent variable occurring in the equation.

Either ψ or θ may be used as the dependent variable and q, K, and A may be expressed in terms of either one. It seems that ψ will be the easier variable to use when experimentally investigating the nature of K. If ψ is a single valued function of θ, the choice is simply a matter of mathematical expediency. The moisture density, or the amount of water in unit volume

of the soil has been used as a variable in expressing capillary flow[5,12], but, for uniform packing this variable is proportional to what is here called the moisture content.

In order to make use of the above equations it is necessary to have information concerning the functions $\theta(\psi)$, and $K(\psi)$. There is already available in the literature[1,3,12,13,14,15] some data showing the relation between θ and ψ for soils and it is possible to determine, for a given porous medium, the way in which the capillary conductivity depends upon the capillary potential.

Experiments with Capillary Conduction

If the pressure gradient and capillary flow are known for a given column of porous medium, the value of the conductivity may be calculated by sub-

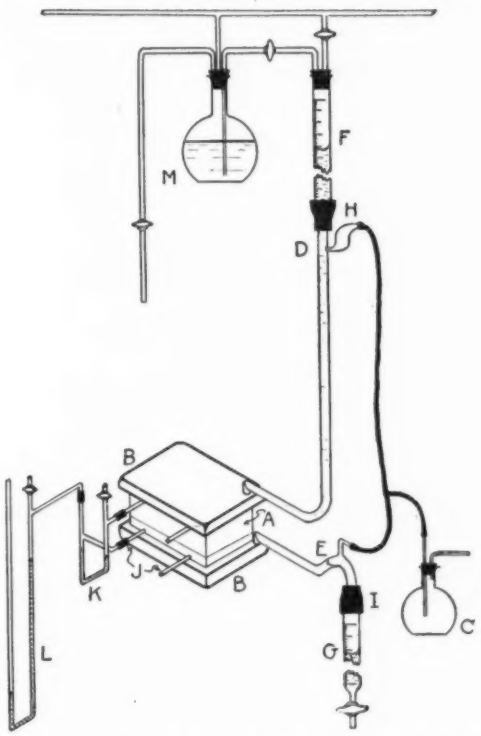

Fig. 2. Apparatus for measuring capillary conductivity.

stituting in Eq. (9). An arrangement of apparatus which has been used in making conductivity measurements is shown in Fig. 2.

A column of soil or other porous medium to be studied is mounted in the rectangular telescoping frame *A* between the two hollow fired clay cells, *B*,

[12] O. W. Israelsen, Hilgardia **2**, 480 (1927).
[13] Livingston, Hemmi and Wilson, Plant Physiol. **1**, 389 (1926).
[14] C. A. Shull, Bot. Gaz. **62**, 1 (1916).
[15] B. J. Korneff, Ann. Sci. Agronom. **43**, 480 (1927).

which have a flat porous wall of uniform thickness in contact with the column of soil. To insure good capillary contact these cells were held against the soil column with strong rubber bands. The cells are filled with water and connected as shown to the vacuum bottle *C* whose pressure is automatically controlled. It is the value of this pressure which determines the moisture content of the column. The pressure in the upper cell is greater than in the lower cell due to the weight of the hydrostatic column from *D* to *E*. With the arrangement shown there is a pressure and a gravity force tending to make the water move from the upper cell through the column to the lower cell. The water flowing in and out of the column is measured by the burettes *F* and *G*. By using pinch clamps at *H* and *I* the upper burette can be filled and the lower one emptied without disturbing the cell water pressure. The pressure difference at the two ends of the column is measured by means of four small porous clay cups *J* extending across the ends of the column. The cups are filled with water and connected to two mercurial manometers, *K*, only one of which is shown in the figure. These cups, which serve as capillary potentiometers, are 5 mm. in diameter and have a wall thickness of about 0.5 mm. Since the pressure drop along lines of flow passing through the cups is small, it is assumed these potentiometers indicate the pressure difference between the two ends of the soil column. If half of this difference is subtracted from the pressure difference indicated by manometer *L*, the average pressure in the water films of the soil column is obtained. Air in the column was at atmospheric pressure and evaporation from the cells and soil was prevented by enclosing them in a paraffined insulite box, the air in which was kept saturated by cloth wicks.

The porous clay cells, having the desired shapes and properties, were made up in the laboratory by the ordinary plaster of Paris casting process.[16] The hollow rectangular cells were cast in two pieces and cemented together with slip when partly dried. The clay used, when fired at 1000°C was readily permeable to water and a wall of the material 0.5 mm thick, when wet, will stand a pressure difference of one atmosphere without leaking air.

The liquid passing through a column was used over and over so that dissolved substances would not continue to change its surface tension and viscosity.

Since the conductivity was to be measured at a constant film thickness or constant curvature it was necessary to maintain a constant difference between the capillary water pressure and atmospheric pressure. Special apparatus[17] was divised for this purpose which would automatically maintain the pressure in flask *C* to within 0.5 mm of mercury of the desired value. Room temperature was controlled by means of a mercury ether thermoregulator[18] and temperature fluctuations as read by a sensitive thermometer were less than 0.1°C. It was found necessary for some of the runs to store the ingoing liquid in a flask, *M*, at reduced pressure so as to prevent

[16] C. F. Binns, The Potter's Craft, New York (1922).
[17] L. A. Richards, Rev. Sci. Inst. **2**, 50 (1931).
[18] L. A. Richards, J.O.S.A. and R.S.I. **18**, 131 (1929).

dissolved air from coming out of solution under the low pressure in the burettes and cells. The connecting tubes were arranged so that any air collecting in the cells would expell itself. Four of the units shown in Fig. 2 were constructed.

When the rate of flow into a column becomes steady and equal to the flow out, a value of the conductivity may be calculated. If Q/t is the inflow in cc per sec., a, the area of the soil column, L its length and Δp the pressure difference, then, multiplying Eq. (9) by the column area to obtain the total flow gives,

$$Q/t = aq = -aK\nabla\Phi. \tag{16}$$

For the value of the water moving force we have

$$-\nabla\Phi = -g - \Delta p/L, \tag{17}$$

the negative signs of the right hand member indicating that the force acts in the negative z direction. Substituting (17) in (16) and solving for K gives

$$K = \frac{Q}{t} \cdot \frac{1}{a(g + \Delta p/L)}. \tag{18}$$

Conductivity data for two soils, A and B, and a ceramic clay C, are given in Table I and Fig. 3. The soil B and clay C are the same as those for which

TABLE I.

Medium	Date	Column area cm^2	Column length cm	Δp dyne cm^{-2}	Q/t cc $sec.^{-1}$	ψ 10^5 erg $gram^{-1}$	K 10^{-11}sec.
Sandy Soil A	May 15	144.0	4.0	0.0	61.0	0.494	415.0
	May 23	144.0	4.0	9750.0	12.1	1.45	24.6
	Oct. 23	144.0	4.0	865.0	160.0	0.201	930.0
	Nov. 11	144.0	4.0	16100.0	11.8	1.35	16.4
	Nov. 22	144.0	4.0	23300.0	6.82	2.38	6.95
Clay Loam Soil B	May 4	151.5	4.4	4740.0	172.5	0.214	555.0
	May 14	151.5	4.4	5630.0	159.0	0.868	465.0
	May 21	151.5	4.4	5920.0	141.0	1.70	400.0
	Oct. 23	151.5	4.4	8440.0	151.5	0.314	341.5
	Oct. 28	151.5	4.4	7650.0	140.0	0.835	340.0
	Nov. 12	151.5	4.4	7900.0	130.0	1.58	309.5
	Nov. 19	151.5	4.4	8970.0	117.6	2.43	257.0
	Dec. 1	151.5	4.4	11000.0	93.8	3.85	178.0
	Dec. 5	151.5	4.4	19600.0	59.8	5.82	72.7
Ceramic Clay C	May 10	112.0	4.5	9080.0	336.0	0.095	1000.0
	May 15	112.0	4.5	34300.0	57.0	0.904	59.0
	May 23	112.0	4.5	40400.0	32.8	1.53	29.4
	Oct. 23	112.0	4.5	16800.0	242.0	0.14	460.0
	Nov. 4	112.0	4.5	53500.0	4.35	1.46	3.28
	Nov. 11	112.0	4.5	28800.0	112.0	0.266	135.0

the curves with the corresponding letters in Fig. 1 were obtained. The A mediums in Figs. 1 and 3 are not the same but seem to have a similar mechanical composition.

In order to follow closely the changes in the rate of flow, readings of the burettes were taken several times a day and except in a few cases when difficulties with the apparatus developed the time intervals between the dates given in Table I indicate the time required for the flow to become steady.

As was stated above, K, which is the flow across unit area in unit time for unit potential gradient, depends on the mechanical composition and state of packing of the medium and on the viscosity and pressure in the water films. For a given determination of the conductivity all of these factors were

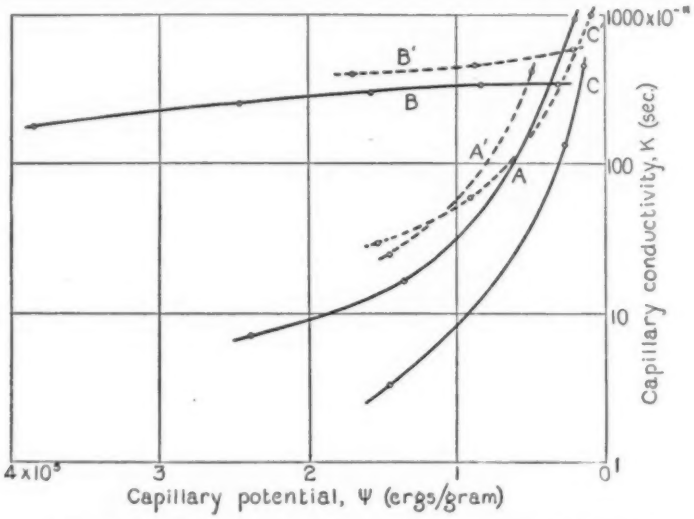

Fig. 3. Capillary conductivity curves for, *A*—a sandy soil, *B*—Greenville loam soil and *C*—Preston clay.

held constant except the state of packing, which, under the compressive stress of the rubber bands, continued to become more dense with time. The effect of this change is indicated in the data.

When the apparatus was first set up the data shown in Fig. 3 as the dotted curves and designated by the primed letters was taken. The apparatus then stood for five months before obtaining the data shown by the full curves. In each case there is a shifting of the curve toward lower values of the conductivity.

Applications of Theory and Data

The curves for the moisture content and conductivity shown in Figs. 1 and 3 have had all the factors except the capillary potential held constant and, before any general application of the flow equations can be made it would be necessary to know something about the relations among ψ, θ and K for different temperatures and different states of packing. However, as an example to illustrate the usefulness of such information, the data given here will be employed in the solution of a typical flow problem.

Let it be required to find for Greenville soil the moisture distribution and steady state upward flow from a water table one meter below the surface.

It will be assumed that the temperature and state of packing correspond to those for which the data were taken. Let us specify the further condition that due to various losses the capillary potential at the surface is $-6.\times10^5$ ergs per gram, this being the lowest value for which the conductivity data are available.

Under the condition of steady linear flow involved in the problem, $\partial\psi/\partial x = \partial\psi/\partial y = \partial\psi/\partial t = 0$ and Eq. (15) becomes

$$K\frac{\partial^2\psi}{\partial z^2} + \frac{\partial K}{\partial z}\frac{\partial\psi}{\partial z} + g\frac{\partial K}{\partial z} = 0. \tag{19}$$

The conductivity data given by curve B, Fig. 3, may be represented analytically by the expression,

$$K = a\psi + b \tag{20}$$

where $a = 5.35\times10^{-15}$ and $b = 3.85\times10^{-9}$. Eq. (19) may now be written

$$(a\psi + b)\frac{\partial^2\psi}{\partial z^2} + a\left(\frac{\partial\psi}{\partial z}\right)^2 + ag\frac{\partial\psi}{\partial z} = 0. \tag{21}$$

The first integration of this equation gives,

$$\frac{\partial\psi}{\partial z} = \frac{1 - gac_1(a\psi + b)}{ac_1(a\psi + b)}. \tag{22}$$

Integrating again to obtain a general solution of (21) we have,

$$\frac{1}{g^2a^2c_1}[1 - gac_1(a\psi + b) - \log(1 - gac_1(a\psi + b))] = z + c_2. \tag{23}$$

If coordinates are chosen such that $z=0$ at the water table, then $\psi=0$ when $z=0$, and $\psi=-6.\times10^5$ when $z=100$. Substituting these conditions in (23) gives two new equations from which the values of the integration constants were found to be, $c_1=-1.66\times10^{19}$ and $c_2=-2288$.

The equation for the flow, under the conditions of the problem, becomes

$$q = -K\nabla\Phi = K(-\partial\psi/\partial z) - g). \tag{24}$$

Substituting the values of K and $\partial\psi/\partial z$ given by Eqs. (20) and (22) gives

$$q = (a\psi + b)\left[\frac{-1 + gac_1(a\psi + b)}{ac_1(a\psi + b)} - g\right] \tag{25}$$

which reduces to

$$q = -1/ac_1 = 1/(1.66\times10^{19}\times5.35\times10^{-15}) = 1.13\times10^{-5} \text{ cc sec.}^{-1}\text{ cm}^{-2} \tag{26}$$

or $q=0.97$ cc per day per cm^2.

Using Eq. (23), the values of ψ for various heights above the water table were calculated and are represented by curve A, Fig. 4. Curve B gives the

moisture content corresponding to the value of the capillary potential at each height under the flow conditions of the problem and was determined by making use of curve *B*, Fig. 1.

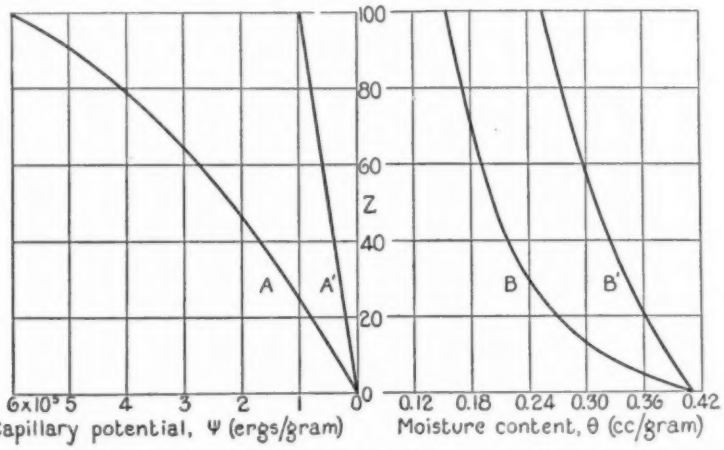

Fig. 4. Curves showing the distribution of capillary potential and moisture content with height above a water table 100 cm below the surface of Greenville soil. *A* and *B* are for upward capillary flow while *A'* and *B'* are for static equilibrium.

The curves *A'* and *B'*, Fig. 4, give the distribution of pressure and moisture with height when there is no surface loss. That is, when the capillary liquid is at rest under gravity and the flow is zero.

Relation between Capillary Potential and Moisture Content

It is of considerable importance in connection with the use of the equations for capillary flow in a porous medium to know, for a certain temperature and state of packing, whether the moisture content is a single valued function of the capillary potential. Haines[19,20] and others[21] have found there is a decided hysteresis effect for a porous medium made up of small spheres. The analogy is made that a medium composed of spheres acts like a capillary tube having a succession of bulges and contractions, because the height to which a liquid stands in such a tube depends on whether the liquid is first raised and allowed to sink to its equilibrium level or whether the liquid rises of itself to the rest position.

If it were possible to set up a system of capillary tubes which would be equivalent to a mass of soil as regards their ability to hold and to transmit moisture in a given direction, it would seem likely that some of these tubes would have a varying cross-section and hence a hysteresis effect might be expected. However, because of the lateral interconnections between soil

[19] W. B. Haines, Jour. of Agric. Sci. **17**, 264 (1927).
[20] W. B. Haines, Jour. of Agric. Sci. **20**, 97 (1930).
[21] Smith, Foote and Busang, Phys. Rev. **37**, 1015 (1931).

capillaries and because the cellular pore spaces between soil particles have various sized openings, it is difficult to predict whether all porous mediums made up of particles will show a hysteresis effect.

The capillary-potential vs. moisture content data shown in Fig. 1 were obtained by having a one half inch layer of soil in contact through a porous plate with water which is at controlled pressure. This method is open to the objections that it is impossible to tell when the soil water is at pressure equilibrium with the water in the cell and also each point on the curve is taken for a different mass of soil. To overcome these undesirable features a fired clay cell like those used in the transmission experiments was suspended on a balance and provided with a lid for covering the upper surface which was porous. Connections to the cell were made by means of a glass tube three meters long and heavy rubber tubing in such a way that when the cell contains water any air accumulating inside will expel itself. The weight of the cell system was made practically independent of changes in the water pressure and a sensitivity of 5 mg in the weighings was attained. When ready for an experiment, a known weight of dry soil is spread on the plate and the amount of moisture in the soil at any later time may be determined from the balance reading.

TABLE II.

	Date	Capillary potential 10^5 erg gram^{-1}	Moisture content cc gram^{-1}
1	Jan. 15	0.072	0.2669
2	" 17	0.930	0.2347
3	" 19	1.50	0.2264
4	" 20	2.93	0.2112
	" 22	6.50	—
5	" 24	4.47	0.1908
6	" 26	2.31	0.2037
7	" 29	0.990	0.2188
8	" 31	0.175	0.2415
9	Feb. 2	0.072	0.2618
10	" 4	0.950	0.2274
11	" 5	1.97	0.2139
12	" 7	0.950	0.2215
13	" 8	0.071	0.2577
14	" 12	0.940	0.2268
15	" 17	2.08	0.2162
16	" 18	3.48	0.2075

The only data yet available with this apparatus were taken for Greenville soil and are presented in Table II and Fig. 5. The cell was filled with soil solution so that transfers of moisture between soil and plate would not cause changes in surface tension and the temperature was controlled for the experiment. Balance readings were taken at frequent intervals after a change in pressure and except for 9 and 13, the points shown represent equilibrium values of the moisture content for the corresponding pressures. It is apparent for a 4 mm layer of soil in a rather loose state of packing there is a considerable hysteresis effect over the pressure range studied.

These results, however, are not conclusive because under the conditions

of the experiment it is probable changes in the state of packing accompanied the changes in moisture content. These changes in packing if present would be in the right direction and might be of sufficient magnitude to account for the observed hysteresis effect.

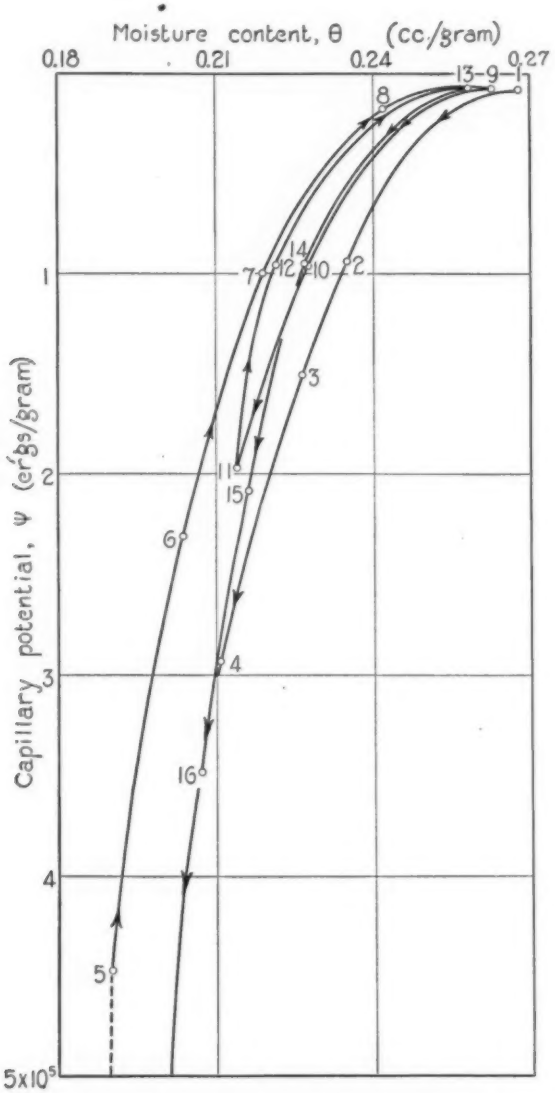

Fig. 5. Curves showing the variation in the moisture content of Greenville soil for cyclic variations in the capillary potential.

To avoid the possibility of the structural change, another experiment using a piece of sandstone as the porous medium has been set up. This material, being composed of loosely cemented particles of various sizes has a rigid structure and should give a good index as to what would happen in a

soil or clay if the pore structure were fixed. As yet no data are available from this experiment.

Another source of information on the hysteresis effect lies in the conductivity experiments. If curves like those shown in Fig. 3 retrace themselves as the potential is increased to zero, it would be good evidence that no hysteresis effect exists. With but one exception the curves shown were obtained by starting with the porous medium wet and proceeding to lower pressures and moisture contents. For curve *C*, as indicated by the dates in Table I, the data for the second point on the curve were taken after the low conductivity determination was made. The consistant relation between the dotted and full curves for the three mediums would seem to indicate that if a hysteresis effect exists for the Preston clay, it does not have a very large effect on the relation between the capillary potential and the conductivity.

The author is indebted to Professor C. C. Murdock for helpful criticism given during the preparation of this paper.

THE THERMAL EXPANSION OF SOME ALUMINUM ALLOYS*

By E. E. McCollough
Department of Chemistry, University of Iowa

(Received September 8, 1931)

THE methods used in determining the thermal expansion of alloys and other metallic solids have passed through a rapid stage of development within recent years. The chief faults of some of the older methods have been lack of accuracy and the requirement of comparatively large samples. The method used in this investigation, known as the "interferometer method" has the advantages of accuracy and of using small specimens. In 1912, Le Chatelier[1] translated a Russian paper on the thermal expansion of annealed aluminum-zinc alloys, by Smirnoff. In 1917 Shakespear[2] made and described some thermal expansion tests on aluminum and several of its alloys. In 1922 Peter Hidnert[3] described an apparatus for accurately determining thermal expansion coefficient. He determined the thermal expansion of copper and some important industrial alloys. In 1925[4] he reported on the thermal expansion of aluminum and various aluminum alloys. The apparatus used was an improvement over that used in his previous work. His bibliography on the subject is very complete serving as a source of most of the references given in this paper.

The present investigation was an attempt to study the effect of alloying elements upon the thermal expansion of aluminum. The effect of varying the percent of a given alloying element and a comparison of the effects of different alloying elements were studied. The types of alloys investigated were aluminum-copper, aluminum-iron, aluminum-magnesium, aluminum-nickel, aluminum-silicon and aluminum-manganese.

Preparation of Samples

The materials for the preparation of the alloys with the exception of some electrolytic copper were furnished by the Aluminum Company of America. A list of the materials used is given below:

Sample No.	Metal	Analysis					
1932	7 A Aluminum	Cu	0.021%,	Fe	0.012%,	Si	0.13%
2418	Commercial aluminum	Cu	0.12%,	Fe	0.24%,	Si	0.13%
1628	Aluminum-iron alloy	Cu	0.04%,	Fe	9.02%,	Si	0.04%
1617	Aluminum-manganese alloy	Fe	0.11%,	Si	13%,	Mn	1.53%
573	Aluminum-silicon alloy	Cu	0.03%,	Fe	0.22%,	Si	18.77%
2340	Aluminum-nickel alloy	Fe	0.10%,	Ni	10%,	Cu	0.05%
	Pure magnesium						
	Electrolytic copper						

[1] Le Chatelier, Comptes Rendus **155**, 351 (1912).

[2] Shakespear, Reports of the Light Alloys Subcommittee, London, p. 48; 1921.

[3] Peter Hidnert, Bull. No. 410, Scientific Papers of the Bureau of Standards.

[4] Peter Hidnert, Bull. No. 497, Scientific Papers of the Bureau of Standards.

* Abstract of a thesis submitted in partial fulfillment of the Doctor of Philosophy Degree in Chemistry at the State University of Iowa.

The aluminum-copper and aluminum-magnesium alloys were prepared by placing the required amount of aluminum in a graphite crucible and melting the material in an electric furnace which had been previously heated to about 800°C. After the aluminum had melted the required amount of alloying material was added and the heating continued for about ten minutes. The molten mixture was poured into a hot graphite mold. The other alloys were prepared by melting the required amount of aluminum and "hardening material" together in the same type of crucible. The samples were milled to a length slightly greater than that required and ground to the proper length on a fine stone or emery paper.

Analytical Methods

All samples of alloys were analyzed to determine the content of alloying elements. The samples for analysis were taken from a portion of the castings as close to the portion used for the thermal expansion determination as possible. Standard analytical methods were used and all determinations made in duplicate. The potassium iodide method[5] was used for the determination of copper. Nickel was determined by the gravimetric dimethylglyoxime method.[6] The aluminum-magnesium alloys were analyzed for magnesium content by the gravimetric magnesium ammonium phosphate method.[7] Iron in the aluminum-iron alloys was determined by the volumetric potassium permanganate method.[8] The method used for the determination of silicon in the aluminum-silicon alloys was a method devised by the Aluminum Company of America.[9] The Volhard method[10] was used in the determination of manganese.

The apparatus used in this investigation was a dental type interferometer, an adaption of the one used by Souder and Peters[11] in their investigation of the expansion of dental materials. In order to control the temperature properly, the interferometer was mounted on a brass block in the bottom of a brass tube about 30 cm long by 6 cm in diameter with 2 mm thickness of walls. The tube was closed with a glass window to prevent radiation. A section of the lower part of the tube was cut away to allow adjustment of the interferometer. The tube was placed on the inside of a steel jacket about one mm thick which was immersed in a light lubricating oil. The oil container was of copper, rectangular in shape and rounded at the corners to eliminate dead spaces. This container was supported in a wooden box and an air space between it and the container retarded heat radiation. A three-junction chromel alumel thermocouple was used to measure the temperature of the sample. The hot-junction was fastened at the bottom of the tube next to the

[5] Scott, Standard Methods of Chemical Analysis, Vol. I, p. 193.
[6] Scott, Standard Methods of Chemical Analysis, Vol. I, p. 333.
[7] Scott, Standard Methods of Chemical Analysis, Vol. I, p. 293.
[8] Popoff, Quantitative Analysis, 3rd edition, p. 138.
[9] Scott, Standard Methods of Chemical Analysis, Vol. I, p. 17.
[10] Scott, Standard Methods of Analysis, Vol. I, p. 302.
[11] Souder and Peters, Technologic Papers of the Bureau of Standards, No. 157; May, 1920

sample. A Brown portable precision potentiometer was used to measure the electromotive force produced by the thermocouple. A thermometer served to measure the temperature of the oil in the bath. The oil bath was cooled by a cooling coil or heated with a resistance coil, the current in which was regulated by a relay operated by a thermostat. The cooling coil could be connected to either a cold brine solution or the tap water supply. The oil in the bath was circulated by a motor driven propeller.

The experimental work performed included the determination of the thermal expansion coefficients from 25° to 100°C of the alloys in the series listed below.

Aluminum-copper alloys

This series includes 10 samples of aluminum-copper alloys containing from 2.3 to 9.4 percent copper. Expansion tests were made from room temperature to about 100°C. Table I gives the chemical analysis and average coefficients of expansion for these alloys.

TABLE I. *Chemical analysis and average coefficients of expansion of aluminum-copper alloys. Temperature range 25 to 100°C.*

Sample number	Percent copper	Average coefficient of expansion per °C	Sample number	Percent copper	Average coefficient of expansion per °C
1	2.30	23.6×10^{-6}	6	7.00	24.2×10^{-6}
2	2.93	23.6	7	7.37	24.4
3	4.27	23.7	8	7.05	24.6
4	6.10	23.8	9	8.87	23.2
5	6.71	24.0	10	9.40	22.6

The thermal expansion data for the aluminum-copper alloys indicate a maximum coefficient at about 8 percent copper. The coefficient decreases sharply after the maximum has been passed. To determine the relations between the constitution of the aluminum-copper alloys and the coefficients of thermal expansion, the literature describing the aluminum end of the aluminum-copper equilibrium diagram[12] was reviewed. Aluminum-rich alloys containing from 0 to 5 percent copper consist of solid solutions of copper in aluminum, and alloys containing from 5 to 33 percent copper consist of solid solutions of $CuAl_2$ in aluminum with an eutectic which contains about 67 percent aluminum. The maximum point in the expansion curve for the aluminum-copper alloys corresponds to about 8 percent copper which is about 3 percent higher than the limit of solubility of copper in aluminum. This investigation did not cover alloys containing more than 11 percent copper, therefore any break in the curve that might occur at the eutectic composition was outside its field.

Aluminum-magnesium alloys

This series includes 8 samples of aluminum-magnesium alloys containing from 1.5 to 15 percent of magnesium. Table II gives the chemical analysis and average coefficients of expansion for these alloys.

[12] Landolt-Börnstein, Physikalisch-Chemische Tabellen, pp. 535–540.

The data for the aluminum-magnesium alloys indicate a minimum coefficient at about 2 percent magnesium content and alloys containing more than 8 percent magnesium have greater coefficients of thermal expansion than that of aluminum. There is no break in the curve of the aluminum-magnesium

TABLE II. *Chemical composition and average of coefficient of expansion of aluminum-magnesium alloys. Temperature range 24–100°C.*

Sample number	Percent magnesium	Average coefficient of expansion per °C	Sample number	Percent magnesium	Average coefficient of expansion per °C.
11	1.52	22.8×10^{-6}	15	8.30	23.9×10^{-6}
12	3.36	23.0	16	10.00	24.2
13	5.98	23.4	17	12.33	24.4
14	7.86	23.7	18	14.94	24.5

temperature composition diagram[13] which corresponds with this minimum. The curve in the equilibrium diagram is almost straight up to about 3.5 percent magnesium and at that point has a sharp break indicating the formation of the compound Al_3Mg_4.

Aluminum-iron alloys

This series of alloys included four samples containing from 2.5 to 11 percent of iron. Table III gives the chemical composition and average coefficients of expansion of this series of alloys.

TABLE III. *Chemical composition and average coefficient of expansion of aluminum-iron alloys. Temperature range 25–100°C.*

Sample No.	Percent iron	Average coefficient of expansion per °C
19	2.50	23.4×10^{-6}
20	6.12	22.8
21	8.82	22.0
22	10.78	21.2

The coefficients of thermal expansion of aluminum-iron alloys decrease with increase of iron content and give a smooth curve through the range of composition investigated. The curve in the aluminum-iron equilibrium diagram[14] does not show any breaks with less than 45 per cent iron.

Aluminum-nickel alloys

This series includes five samples of aluminum-nickel alloys. Thermal expansion tests were made as for the previous series. The results are given in Table IV.

[13] Landolt-Börnstein, Physikalisch-Chemische Tabellen, pp. 535–540.
[14] Landolt-Börnstein, Physikalisch-Chemische Tabellen, pp. 535–540.

TABLE IV. *Chemical composition and average coefficient of expansion of aluminum-nickel alloys. Temperature range 25–100°C.*

Sample No.	Percent nickel	Average coefficient of expansion per °C
23	2.75	23.4×10^{-6}
24	4.50	23.6
25	5.78	23.2
26	7.24	22.8
27	8.22	22.4

The thermal expansion data for the aluminum-nickel alloys indicate that the coefficient of expansion decreases slightly with increase of nickel over the range of compositions investigated.

Aluminum-silicon alloys

Four samples of aluminum-silicon alloys were investigated. The percent silicon ranged from 4 to 16. Thermal expansion tests were performed in the same manner as for the other series. The coefficients of expansion and chemical composition are given in Table V.

TABLE V. *Average coefficients of expansion and composition of aluminum-silicon alloys. Temperature range 25–100°C.*

Sample No.	Percent silicon	Average coefficient of expansion per °C
28	4.22	22.4×10^{-6}
29	9.13	21.8
30	12.20	20.2
31	16.10	19.2

The coefficients of thermal expansion of aluminum-silicon alloys decreases with increase of silicon content. Silicon had a greater effect on the thermal expansion of aluminum than any of the other alloying elements investigated.

Aluminum-manganese alloys

This series includes three samples of aluminum-manganese alloys. The percent manganese varied from 0.5 to 1.5. Table VI gives the thermal expansion and chemical composition of the alloys.

TABLE VI. *Average coefficients of expansion and chemical composition of aluminum-manganese alloys. Temperature range 25–100°C.*

Sample No.	Percent manganese	Average coefficient of expansion per °C
32	0.52	23.7×10^{-6}
33	1.12	23.7
34	1.48	23.4

Only three alloys of aluminum-manganese were investigated as commercial "hardening materials" containing more than 1.5 percent manganese were not available. The addition of 1 to 1.5 percent manganese causes a decrease

in the thermal expansion of aluminum. Aluminum-manganese alloys containing about 1.5 percent manganese have lower coefficients of expansion than those containing 0.5 to 1 percent manganese.

Aluminum-copper-silicon-manganese alloys

This series involved five samples of aluminum alloys obtained from the Aluminum Company of America. Thermal expansion tests were made from room temperature to about 100°C. The chemical compositions and the average coefficients of expansion are given in Table VII.

TABLE VII. *Chemical composition and average coefficients of expansion of aluminum-silicon-copper-manganese alloys.*

Sample No.	Chemical composition*			Temp. range	Average coefficient of expansion per °C
	Cu %	Si %	Mn %		
35	4.00	0.30	0.55	25–100°C	22.4×10^{-6}
36	4.14	0.22	0.59	25–100°C	22.6
37	4.37	0.74	0.81	25–100°C	22.8
38	0.14	0.36	1.07	25–100°C	23.2
39	0.11	0.38	1.29	25–100°C	23.1

* Chemical analysis made by Aluminum Company of America on all samples of this series.

The coefficients of expansion found for alloys of aluminum with copper, manganese and silicon compare favorably with those of Hidnert.[3]

Attempts were made to derive an empirical equation by which the coefficient of thermal expansion of alloys could be calculated on the assumption that the thermal expansion should be the arithmetical mean of the coefficients of the alloying elements. This was found to hold roughly over the smooth parts of the curves but did not hold for portions including maximum and minimum points. Silicon-aluminum and silicon-manganese alloys were found to apply more closely than the others. No general relations were found between the percentages of the alloying elements and the corresponding coefficients of thermal expansion of the alloys.

CONCLUSIONS

All of the alloying elements investigated have an effect on the thermal expansion of aluminum. An empirical equation based upon the assumption that the coefficient of thermal expansion of the alloy is the arithmetical mean of the coefficients of expansion of aluminum and the alloying element, is limited in its use.

THE EDITOR'S COLUMN

The Spinor Calculus

IT IS an event of prime importance in the development of a science when a discovery is made in one of its branches which is of fundamental interest in another branch. This interplay between related fields is one of the most fruitful sources of inspiration for the working scientist—without it he would be in grave danger of becoming "stale."

The quantum theory can be credited with the occurrence of at least one such event which is of importance to the theory of relativity. In the special theory of relativity the so-called 4-vectors or "world" vectors play a fundamental role. The use of directed line segments or vectors (sometimes called "arrows") for the representation of ordinary physical quantities such as force, velocity, acceleration, etc., is familiar to every student of elementary physics. In the four-dimensional space of the relativity theory in which there is a fusion of the ordinary space of three dimensions with a fourth coordinate representing a "time coordinate," there are analogous quantities which are called 4-vectors, as mentioned above. The important feature about vectors in two, three, four, or in (mathematical) space of any number of dimensions is that they are, in a sense, independent of the choice of the coordinate system which is used for the description of the happenings within the system.[1] For example, the magnitude and direction of the force acting on a particle are independent of how we choose a set of axes for the measurement of the components of the force. Since a directed line segment or arrow has just this same independence of the coordinate system; i.e. an independence of magnitude and direction, it is convenient to represent a force by such an arrow. The situation in four-dimensional space is quite analogous.

On the other hand there are certain types of physical quantity which are not representable by vectors, but which nevertheless have a significance independent of any choice of coordinate system. An example of such a quantity is the stress within an elastic medium when it is in a strained condition. At a given point within the material one imagines a small plane surface element with its normal oriented in a specified direction. The force which the material on one side of the surface exerts upon that on the other side is a vector, but the magnitude and direction of this vector depend upon the orientation of the surface element. In order completely to specify the stress at a point within the material one must be able to give the force acting across a plane element of surface oriented in any way. Stress thus has a more extended character than does a simple force or velocity, for while a statement of the three components of a force or velocity in a given coordinate system will completely determine the force or velocity, it is found necessary to specify nine quantities in order to determine the stress at a point. While not representable by a vector, the stress is representable by another type of mathematical quantity called a "tensor," which has nine instead of three components. This ratio of nine to three for the ratio of the number of components of a tensor to the number of components of a vector is not accidental, for if one multiplies together the components of two vectors, taking one component of each vector, the resulting $3\times3=9$ quantities will form the components of a tensor. Such a tensor formed by the multiplication of two arbitrary vectors will in general not represent any particular physical quantity. In four-dimensional space a vector has four components while a tensor has $4\times4=16$ components.

It was a part of the program of the theory of relativity to express all of the equations of physics as equations between 4-vectors or tensors, because if this were done then the *form* of the equations would be the same for every coordinate system of reference. A very simple example of such invariance of form is the force equation (in vector form) of elementary mechanics

$$F=ma.$$

This program was based on the definite supposition that no equations between physical

[1] Cf. A. Kopff, "The Mathematical Theory of Relativity" for details.

quantities would be discovered which could not be written as equations connecting tensors, or as one says, "in tensor form."

IT WAS at just this point that the developments in quantum theory were brought to bear. Through the discovery of his famous wave equation for one electron by Dirac[2] in 1928 an example was found of an equation which is consistent with the special theory of relativity and yet which is not in tensor form. Dirac's efforts to combine the quantum theory with the theory of relativity have been fruitful of results for both theories.

Many attempts have been made to put Dirac's equation in tensor form,[3] but it appears to be difficult to accomplish the reduction successfully. This has lead to the idea that maybe, after all, vectors and tensors are not the fundamental units in terms of which physical quantities should be expressed, and as a result this important property has now been assigned to a new type of quantity, the "spinor". While a 4-vector has four, and a "4-tensor" has sixteen components, a spinor has only two. From two spinors one can compose a 4-vector by multiplying together the components of the spinors taken two at a time and combining the products according to a definite rule. If one uses two arbitrary spinors then in general he cannot form a 4-vector from the products of their components, but these products form instead what are called the components of a spinor of the second rank —only certain kinds of spinors of the second rank can be related to 4-vectors.

The spinor calculus was developed in 1929 by the Dutch mathematician van der Waerden.[4] It is reported that an equivalent theory was worked out a number of years ago by a French mathematician, but references to it are not known to the writer. Recently Laporte and Uhlenbeck[5] have published a very interesting account of the theory and its applications to Dirac's equation, and also to Maxwell's electromagnetic equations. The four "wave functions" appearing in Dirac's equation are found to correspond to the components of two spinors of the first rank, so that Dirac's equation is a spinor and not a tensor equation. Maxwell's equations can also be put in spinor form by using spinors of the second rank. Oppenheimer has lately employed the spinor calculus in his study of the theory of light quanta.[6]

One of the difficulties in the spinor analysis is that while one can construct vectors and tensors from spinors, one cannot in general unravel a vector into the proper related spinors of first rank. Lacking a general physical interpretation for spinors themselves, one is hampered in the setting up of the proper spinor equations. This is a problem for the future, but in any event the spinor seems destined to play an important part not only in physical theory, but perhaps also in the theory of relativity itself.

E. L. HILL

[2] P. A. M. Dirac, Proc. Roy. Soc. A, Feb. and March, (1928).

[3] E. G. V. Fock, Zeits. f. Physik **57,** 261 (1929); Jour. de Physique **10,** 392 (1929). A. S. Eddington, Proc. Roy. Soc. **A121,** 524 (1928). J. M. Whittaker, Cam. Phil. Soc. **24,** 501 (1928). B. Podolsky, Phys. Rev. **37,** 1398 (1931).

[4] B. van der Waerden, Gottinger Nachrichten, (1929).

[5] O. Laporte and G. Uhlenbeck, Phys. Rev. **37,** 1380 and 1552 (1931).

[6] J. R. Oppenheimer, Phys. Rev. **38,** 725 (1931).

The Production of High Velocity Charged Particles

IN A number of physical laboratories in America and in other countries concerted efforts are being made to develop methods for the production of charged particles, that is, electrons, protons, or even heavier ions, with very high velocities. To the physicist bent on exposing the innermost secrets of the atom these experiments are of great interest. At the present time the only sources of very high speed electrons or nuclei are the radio-active elements. Radioactive substances are available only in small amounts, so that the sources are weak, and furthermore one has no control over the speed with which a charged particle is emitted from the nucleus, so that to obtain particles with varying velocities one must employ a series of sources or else slow down the particles by interposing an absorbing medium in the path. While a great amount of valuable work has been done by this means, it would be very helpful to have available a strong source of particles whose velocities could be varied more or less at will. With such a source one could make a powerful attack on the problem

of the artificial disintegration and the building-up of nuclei.

Breit, Tuve, Dahl, and Hafstad[1] have developed a method for applying the high voltages developable by means of Tesla coils to x-ray tubes. The tubes are of a curious design consisting of a number of sections, with the anode and cathode of the order of a meter apart. Such tubes were used previously by W. D. Coolidge. With the tube operating at 1,300 kilovolts the "x-rays" emitted were found to be about as hard as the γ-rays from radium. Some preliminary measurements have also been made on the cathode rays.

Lauritsen[2] and his co-workers have developed an x-ray tube operated by a set of transformers in cascade. Instead of making the tube in sections, as was done in the experiments of Breit and his co-workers, the electrodes were placed very close together, this tending to prevent stray electrons from bombarding the glass walls of the tube, causing it to disintegrate. They have been able to operate the tube on 750 kilovolts.

A very spectacular set of experiments is being conducted by A. Brasch and F. Lange[3] of Berlin. They have proposed to use as a source of high potential the tremendous fields which exist in the atmosphere during storms. They suspended a collector system in a small valley near Mt. Generoso in Italy, a region which is subject to thunderstorms. The suspension system was heavily insulated and the observers were stationed in a metallic house for protection. They have obtained sparks of a length which were estimated to be due to potential differences of 8 to 15 million volts. The authors have also been studying laboratory methods of producing high potentials using condensor systems, and have been able to produce x-rays with an exciting potential of around 2 million volts, and positive rays of hydrogen with an accelerating potential of nearly a million volts.

ALMOST ingenious method is being used successfully by E. Lawrence and M. Livingston[4] for the production of protons and hydrogen molecular ions with energies corresponding to accelerating potentials in excess of a half million volts. The idea of the method is very simple: two circular metallic plates are joined by another piece of metal so that the whole forms a closed right cylinder of small height. The whole is cut into two parts along a diameter of the plates, and the two halves separated by a small distance, the two parts being connected to an alternating voltage generator developing a potential difference of around 2000 volts. The system is placed in an uniform magnetic field perpendicular to the plane of the plates. Electrons and protons are then formed near the center of the cylinder in the space between the two sections, and under the influence of the potential difference between sections are accelerated into one or other of the halves of the cylinder. Due to the presence of the magnetic field they begin to spiral about the lines of force and are thus caused to pass again across the gap between the sections. Those particles which get in phase with the alternating field across the gap will continue to spiral around the lines of force of the magnetic field, falling through a potential difference of about 2000 volts every time they pass across the gap. As the velocity increases, the radius of the path also increases so that ultimately it will be as large as the radius of the plates at which time the particles can be collected in a collecting cage, or could be admitted to another piece of apparatus through a slit system. By using a larger magnet and a higher frequency on the alternating voltage generator the authors hope to be able to obtain protons with an energy corresponding to a fall through a potential drop of 10 million volts.

Lawrence and Sloan[5] have also reported on a method for obtaining high speed ions by shooting them down the axis of a column of hollow cylinders which are connected alternately to the poles of a high frequency oscillator. They have been able to produce singly charged mercury ions with speeds equivalent to that obtained by a fall through a potential difference of over a million volts.

E. L. HILL

[1] G. Breit, M. A. Tuve, and O. Dahl, Phys. Rev. **35,** 51 (1930); M. A. Tuve, G. Breit, and L. R. Hafstad, Phys. Rev. **35,** 66 (1930); M. A. Tuve, L. R. Hafstad, and O. Dahl, Phys. Rev. **37,** 469 (1931).

[2] C. C. Lauritsen and B. Cassen, Phys. Rev. **36,** 988 (1930); C. C. Lauritsen, Phys. Rev. **36,** 1680 (1930).

[3] A. Brasch and F. Lange, Zeits. f. Physik **70,** 10 (1931).

[4] E. Lawrence and M. S. Livingston, Phys. Rev. **38,** 834 (1931).

[5] E. Lawrence and D. Sloan, Phys. Rev. **38,** 586 (1931).